A Distributional Framework for Matched Employer Employee Data*

Stéphane Bonhomme, University of Chicago

Thibaut Lamadon, University of Chicago

Elena Manresa, New York University

Revised Draft: October 2018

Abstract

We propose a framework to identify and estimate earnings distributions and worker composition on matched panel data, allowing for two-sided worker-firm unobserved heterogeneity and complementarities in earnings. We introduce two models: a static model that allows for nonlinear interactions between workers and firms, and a dynamic model that allows in addition for Markovian earnings dynamics and endogenous mobility. We show that this framework nests a number of structural models of wages and worker mobility. We establish identification in short panels, and develop tractable two-step estimators where firms are classified in a first step. Applying our method to Swedish administrative data, we find that log-earnings are approximately additive in worker and firm heterogeneity. Our estimates imply the presence of strong sorting patterns between workers and firms, and a small contribution of firms – net of worker composition – to earnings dispersion. In addition, we document that wages have a direct effect on mobility, and that, beyond their dependence on the current firm, earnings after a job move also depend on the previous employer.

JEL codes: J31, J62, C23.

Keywords: two-sided heterogeneity, bipartite networks, matched employer employee data, sorting, job mobility.

*First draft: November 2014. We thank the co-editor and four anonymous referees, our discussants Pat Kline and Chris Hansen, and Joe Altonji, Bryan Graham, Rafael Lopes de Melo, Costas Meghir, Luigi Pistaferri, Jean-Marc Robin, Adam Rosen, Raffaele Saggio, Uta Schoenberg, Robert Shimer, Chris Taber, and audiences at various places for useful comments. We thank the IFAU for access to, and Lisa Laun for help with, the Swedish administrative data. The authors acknowledge support from the NSF grant number SES-1658920.

Contact: sbonhomme@uchicago.edu; lamadon@uchicago.edu; em1849@nyu.edu.

1 Introduction

Identifying the contributions of worker and firm heterogeneity to earnings dispersion is an important step towards answering a number of economic questions, such as the nature of sorting patterns between heterogeneous workers and firms or the sources of earnings inequality.

Two influential literatures have approached these questions from different angles. The method of Abowd, Kramarz, and Margolis (1999) (AKM hereafter) relies on two-way fixed-effect regressions to account for unobservable worker and firm effects, and allows quantifying their respective contributions to earnings dispersion and correlations between worker and firm heterogeneity. The AKM method is widely used in labor economics and outside.[1] A second literature tackles similar issues from a structural perspective, by developing and estimating fully specified theoretical models of sorting on the labor market.[2]

Reconciling these reduced-form and structural literatures has proven difficult. While the AKM method provides a tractable way to deal with two-sided unobserved heterogeneity, the AKM model relies on substantive, possibly restrictive assumptions. The absence of interactions between worker and firm attributes restricts complementarity patterns in earnings. This is at odds with numerous theories which, since Gary Becker's work, have emphasized the link between complementarity and sorting (Shimer and Smith, 2000, Eeckhout and Kircher, 2011). In addition, the AKM model is static, in the sense that worker mobility does not depend on earnings realizations conditional on worker and firm heterogeneity, and that earnings after a job move do not depend on the previous firm. Such a static model is not able to account for a number of mechanisms that have been emphasized in the dynamic structural literature.

On the other hand, attempts at structurally estimating dynamic models of sorting have faced computational and empirical challenges. The dimensions involved are daunting: how to estimate a model of worker mobility and earnings with hundreds of thousands of workers and dozens of thousands of firms in the presence of both firm and worker unobserved heterogeneity? And how informative are functional form assumptions in these often tightly parameterized models?

[1] Applications of the method to earnings data include Gruetter and Lalive (2009), Mendes et al. (2010), Woodcock (2008), Card et al. (2013), Goldschmidt and Schmieder (2015), Song et al. (2015), and Sorkin (2018), among others. The AKM estimator has been used in a variety of other fields, for example to link banks to firms or teachers to schools or students, or to document differences across areas in patients' health care utilization (e.g., Kramarz et al., 2015, Jackson, 2013, Finkelstein et al., 2016).

[2] Many structural models proposed in the literature build on Becker (1973). Examples are Lopes de Melo (2018), Lise et al. (2016), Bagger and Lentz (2014), Hagedorn et al. (2017), and Lamadon et al. (2013).

In this paper we introduce an empirical framework with two-sided unobserved heterogeneity that nests a range of theoretical mechanisms emphasized in the literature. While allowing for rich patterns of complementarities, sorting, and dynamics, the framework preserves parsimony using a dimension reduction technique to model firm heterogeneity. We propose two models, static and dynamic, which allow for interaction effects between worker and firm heterogeneity. In the dynamic model we let job mobility depend on earnings realizations in addition to worker and firm attributes, and we allow earnings after a job move to depend on attributes of the previous firm beyond those of the current one. Dynamic persistence is specified as first-order Markov.

We provide conditions for identification in short panels under discrete worker heterogeneity. The primary source of identification is given by job movers. For the static model we rely on two periods, while we use four periods to identify the dynamic model. The ability of our method to deal with short panels is important, since assuming time-invariant heterogeneity of either workers or firms over long periods may be unattractive. Our analysis shows that mobility and heterogeneity patterns play a key role in identifying complementarities.

We define the relevant level of firm unobserved heterogeneity as the *class* of a firm. We model worker types as draws from a discrete distribution, and allow for unrestricted interactions between worker types and firm classes. In principle, a class could be a firm itself. However, in typical matched employer employee data sets the number of job movers per firm tends to be small, which creates an incidental parameter bias in estimation.[3] In such environments, reducing the number of classes can alleviate small-sample biases. We use a k-means clustering estimator to classify firms based on how similar their earnings distributions are. The classification may also be based on mobility patterns or longitudinal earnings information, and it can be modified to incorporate firm characteristics such as value added. We establish the consistency of the classification under discrete firm heterogeneity.[4]

We use a two-step approach for estimation. In the *classification* step we group firms into classes using k-means clustering, and in the *estimation* step we estimate the model by allowing for firm class heterogeneity. We estimate the model by maximum likelihood, conditional on the estimated firm classes. This asymmetric treatment of firm and worker heterogeneity is

[3]See Abowd et al. (2004), Andrews et al. (2008, 2012), and recently Kline et al. (2018) for methods to address incidental parameter bias in fixed-effects regressions.

[4]Similarly as in most of the literature on discrete estimation, this result is derived under the assumption that the population of firms consists of a finite, known number of classes. In Bonhomme et al. (2017) we consider a setting where the discrete modeling is viewed as an approximation to an underlying, possibly continuous, distribution of firm unobserved heterogeneity, and we provide consistency results and rates of convergence.

helpful for tractability. We verify in simulations that our estimator performs well in data sets similar to the one of our application. We also confirm the ability of our estimator to recover wage functions in data sets generated according to the theoretical model of Shimer and Smith (2000), extended to allow for on-the-job search, under both positive and negative assortative matching. Finally, we also develop simple estimators for regression counterparts to our static and dynamic models, and use them to show the robustness of our main results.

We take our approach to Swedish matched employer employee panel data, focusing on males for the 2002-2004 period. The estimates of our static model imply that an additive model provides a good first-order approximation to the variance structure of log-earnings, although our results also highlight the presence of some complementarities between firms and lower-type workers. Between-firm differences explain 38% of the overall log-earnings variance. However our estimates imply that, net of the effect of worker composition, firm heterogeneity accounts for less than 5% of the overall variance (that is, less than 13% of the between-firm variance). In addition, we find a strong association between worker and firm heterogeneity, with a correlation ranging between 30% and 50% depending on the specification. The largest share of the variance is explained by worker heterogeneity.

These results suggest that similar workers are not paid very differently across employers, although different workers tend to work in very different firms. The presence of strong sorting, together with the absence of strong complementarities in wages, are hard to reconcile with models where sorting is driven by complementaries in production, as in Becker (1973). Alternative explanations for sorting have been proposed, such as the presence of amenities, peer effects or more complex heterogeneity, although our findings might also be partly driven by specificities of the Swedish labor market.

The estimates of our dynamic model on the 2001-2005 period, besides being in line with the cross-sectional variance decomposition implied by our static model, shed light on several mechanisms that have been emphasized in the structural literature. In particular, we find that low earnings realizations, conditional on worker and firm heterogeneity, tend to make workers more likely to move. This failure of the strict exogeneity assumption may indicate the presence of match heterogeneity. We also find evidence of an effect of the previous employer on current earnings, conditional on the current firm's class. This state dependence effect could be rationalized by existing theories, such as the offer and counteroffer mechanism of Postel-Vinay and Robin (2002).

Literature and outline. The methods we propose contribute to a large literature on the identification and estimation of models with latent heterogeneity. Discrete fixed-effects approaches have recently been proposed in single-agent panel data analysis (Hahn and Moon, 2010, Lin and Ng, 2012, Bonhomme and Manresa, 2015). The k-means clustering algorithm we use to classify firms is widely used in a number of fields, and efficient computational routines are available (Steinley, 2006). Here we use such an approach in models with two-sided heterogeneity. Our approach to identification and estimation of mixture models has a number of precedents in the literature, such as Hall and Zhou (2003), Hu (2008), Henry et al. (2014), Levine et al. (2011), Bonhomme et al. (2014), and Hu and Schennach (2008) and Hu and Shum (2012) for continuous mixtures. Our conditional mixture approach is also related to mixed membership models (Blei et al., 2003, Airoldi et al., 2008).

Compared to this previous work, we rely on a hybrid "one-sided correlated random-effects" approach that models the firm classes as discrete fixed-effects and the worker types as (discrete or continuous) random-effects correlated with the firm classes. This approach is motivated by the structure of typical matched employer employee data sets. With sufficiently many workers per firm, firm class membership will be accurately estimated. In contrast, the number of observations for a given worker is typically small. This approach can alleviate the incidental parameter bias of fixed-effects estimators, particularly in short panels. It also offers a tractable way of allowing for complementarities and dynamics. Bonhomme (2017) reviews existing econometric methods for bipartite network data.

Also related, Abowd et al. (2018) propose a Bayesian approach where both firm and worker heterogeneity are discrete. Their setup allows for latent match effects to drive job mobility, in a way that is related to, but different from, our dynamic model. Hagedorn et al. (2017) propose to recover worker types by ranking workers by their earnings within firms, and aggregating those partial rankings across firms. Their method relies on long panels, and exploits the implications of a structural model to identify firm heterogeneity. In contrast, while our framework nests a number of theoretical models of wages and mobility it is not tied to a specific structural model.

The outline of the paper is as follows. In Section 2 we present the framework. In Sections 3 and 4 we study identification and estimation. In Sections 5 and 6 we show empirical results based on the static and dynamic models. Lastly, we conclude in Section 7. A supplementary appendix with additional results can be found on the authors' web pages.

2 Framework of analysis

We consider an economy composed of N workers and J firms. We denote as j_{it} the identifier of the firm where worker i is employed at time t. Job mobility between a firm at t and another firm at $t+1$ is denoted as $m_{it} = 1$.

Heterogeneity across firms is characterized by their *class*. We denote as $k_{it} = k(j_{it})$ in $\{1, ..., K\}$ the class of firm j_{it}. Classes form a partition of the set of firms into K classes. There may be as many classes as firms, in which case $K = J$ and $k_{it} = j_{it}$. Alternatively, firm classes could be defined in terms of observables such as industry or size. In Section 4 we describe a method to consistently estimate the latent classes k_{it} from the data, under the assumption that firm heterogeneity has a finite, known number of points of support in the population.

Workers are also heterogeneous, and we denote the *type* of worker i as α_i. These types can be discrete or continuous, depending on the model specification. In addition to their unobserved types, workers may also differ in terms of their observable characteristics X_{it}.

Lastly, worker i receives log-earnings Y_{it} at time t. The observed data for worker i is thus a sequence of earnings $(Y_{i1}, ..., Y_{iT})$, firm and mobility indicators $(j_{i1}, m_{i1}, ..., j_{i,T-1}, m_{i,T-1}, j_{iT})$, and covariates $(X_{i1}, ..., X_{iT})$. We consider a balanced panel setup for simplicity, and we focus on workers receiving positive earnings in each period.

In this framework we will be interested in recovering the distributions of log-earnings for workers of type α in firms of class k, and the proportions of type-α workers in class-k firms. Earnings distributions will be informative about complementarities, while type proportions will be informative about sorting patterns. In addition, within our framework we will be able to document transition probabilities and other dynamic aspects.

We consider two different models: a static model where current earnings do not affect job mobility or future earnings conditional on worker type and firm class, and a dynamic model that allows for these possibilities. We now describe these two models in turn. Next we discuss how our assumptions map to theoretical sorting models proposed in the literature. Throughout we denote $Z_i^t = (Z_{i1}, ..., Z_{it})$ the history of a random variable Z_{it} up to period t.

2.1 Static model

There are two main assumptions in the static model. First, job mobility may depend on the type of the worker and the classes of the firms, but not directly on earnings. As a result, the firm and mobility indicators, and firm classes, are all *strictly exogenous* in the panel data sense. In addition, covariates are also strictly exogenous. Second, log-earnings after a job move are not allowed to depend on previous firm classes or previous earnings, conditional on the worker type and the new firm's class.

Before stating the assumptions formally let us describe the model's timing. In period 1 the type of a worker i, α_i, is drawn from a distribution that depends on the class k_{i1} of the firm where she is employed and her characteristics X_{i1}. The worker draws log-earnings Y_{i1} from a distribution that depends on α_i, k_{i1}, and X_{i1}.

At the end of every period $t \geq 1$, the worker moves to another firm (that is, $m_{it} = 1$ or 0) with a probability that may depend on her type α_i, her characteristics X_i^t, the fact that she moved in previous periods m_i^{t-1}, and current and past firm classes k_i^t. This probability, like all other probability distributions in the model, may depend on t unrestrictedly. Moreover, the probability that the class of the firm she moves to is $k_{i,t+1} = k'$ may also depend on α_i, X_i^t, m_i^{t-1}, and k_i^t (while also varying with k'). Lastly, covariates $X_{i,t+1}$ are drawn from a distribution depending on α_i, X_i^t, m_i^t, and k_i^{t+1}.

If the worker changes firm (that is, when $m_{it} = 1$), log-earnings $Y_{i,t+1}$ in period $t+1$ are drawn from a distribution that depends on α_i, $X_{i,t+1}$, and $k_{i,t+1}$. If instead the worker remains in the same firm between t and $t+1$ (that is, $m_{it} = 0$), $Y_{i,t+1}$ are drawn from an unrestricted distribution that may depend on Y_i^t, α_i, X_i^{t+1}, and k_i^{t+1}.

Formally the two main assumptions are thus as follows.

Assumption 1. *(static model)*

(i) (mobility determinants) m_{it}*,* $k_{i,t+1}$ *and* $X_{i,t+1}$ *are independent of* Y_i^t *conditional on* α_i*,* k_i^t*,* m_i^{t-1}*, and* X_i^t*.*

(ii) (serial independence) $Y_{i,t+1}$ *is independent of* Y_i^t*,* k_i^t*,* m_i^{t-1} *and* X_i^t *conditional on* α_i*,* $k_{i,t+1}$*,* $X_{i,t+1}$*, and* $m_{it} = 1$*.*

A simple example of the static model is the following log-earnings regression:

$$Y_{it} = a_t(k_{it}) + b_t(k_{it})\alpha_i + X_{it}'c_t + \varepsilon_{it}, \tag{1}$$

where $\mathbb{E}\left(\varepsilon_{it} \,|\, \alpha_i, k_i^T, m_i^T, X_i^T\right) = 0$. This model simplifies to the AKM model in the absence of interaction effects, i.e. when $b_t(k) = 1$, and firms j_{it} and classes k_{it} coincide.[5]

2.2 Dynamic model

There are two main differences between the dynamic and static models. First, at the end of period t the worker moves to another firm with a probability that depends on her current log-

[5]While both parts in Assumption 1 are needed to identify the full model, restrictions on dependence are not needed to identify parameters such as $a_t(k)$, $b_t(k)$ and c_t in (1).

earnings Y_{it} in addition to her type α_i, X_{it}, and k_{it}, and likewise the probability to move to a firm of class $k_{i,t+1} = k'$ also depends on Y_{it}. Second, log-earnings $Y_{i,t+1}$ in period $t+1$ are drawn from a distribution depending on the previous log-earnings Y_{it} and the previous firm class k_{it}, in addition to α_i, $X_{i,t+1}$, and $k_{i,t+1}$. Job movers and job stayers draw their log-earnings from different distributions conditional on these variables. As we discuss in the next subsection, allowing for these features is important in order to nest a number of structural models of wage and employment dynamics that have been proposed in the literature. Formally we make the following assumptions.

Assumption 2. *(dynamic model)*

(i) (mobility determinants) m_{it}, $k_{i,t+1}$ *and* $X_{i,t+1}$ *are independent of* Y_i^{t-1}, k_i^{t-1}, m_i^{t-1} *and* X_i^{t-1} *conditional on* Y_{it}, α_i, k_{it}, *and* X_{it}.

(ii) (serial dependence) $Y_{i,t+1}$ *is independent of* Y_i^{t-1}, k_i^{t-1}, m_i^{t-1} *and* X_i^t *conditional on* Y_{it}, α_i, $k_{i,t+1}$, k_{it}, $X_{i,t+1}$, *and* m_{it}.

Assumption 2 consists of two first-order Markov conditions. In part (i), log-earnings Y_{it} are allowed to affect the probability to change job directly between t and $t+1$, but the previous earnings $Y_{i,t-1}$ do not have a direct effect.[6] Similarly, in part (ii) log-earnings $Y_{i,t+1}$ may depend on the first lag of log-earnings Y_{it}, and on the current and lagged firm classes $k_{i,t+1}$ and k_{it}, but not on the further past such as $Y_{i,t-1}$ and $k_{i,t-1}$. Also note that, unlike in the static model, Assumption 2 (ii) restricts the evolution of log-earnings within as well as between jobs.

As a simple dynamic extension of (1) one may consider the following specification for the earnings of job movers between $t-1$ and t (i.e., $m_{i,t-1} = 1$):

$$Y_{it} = \rho_t Y_{i,t-1} + a_{1t}(k_{it}) + a_{2t}(k_{i,t-1}) + b_t(k_{it})\alpha_i + X_{it}'c_t + v_{it}, \tag{2}$$

where $\mathbb{E}\left(v_{it} \,|\, \alpha_i, k_i^t, m_i^{t-1}, Y_i^{t-1}, X_i^t\right) = 0$. Here log-earnings after a job move may depend on earnings and firm class in the previous job.

2.3 Links with theoretical models

In this subsection we study whether our assumptions are compatible with various theoretical models of the labor market. We consider models that abstract from hours of work, so we refer to earnings and wages indistinctively.

[6] Assumption 2 (i) allows $X_{i,t+1}$ to be drawn from a distribution that depends on Y_{it} as well as α_i, X_{it}, m_{it}, and $k_{i,t+1}$. Our identification arguments apply to this case, and estimation could allow for predetermined individual characteristics, such as job tenure.

Models where the relevant state space is (α, k_t). We first consider models where wages are a function, possibly non-linear or non-monotonic, of the worker type α, the firm class k_t, and a time-varying effect, say ε_t, where ε_t does not affect mobility decisions. This structure is compatible for instance with wage posting models (as in Burdett and Mortensen, 1998, Delacroix and Shi, 2006, or Shimer, 2005), where the wage paid to a worker does not have any history dependence and ε_t is classical measurement error or an i.i.d. match effect realized after mobility. This means that, while allowing for rich mobility and earnings patterns, such models are compatible with the assumptions of our static model, see Assumption 1.

Similarly, Assumption 1 is compatible with models where the wage is set as the outcome of a bargaining process between the firm and the worker under certain conditions on the worker's outside option. For example, this is the case in Shimer and Smith (2000), where the outside option is unemployment since workers always go through unemployment before finding a new job; see also Hagedorn et al. (2017). In such sorting models, specifying the wage function in a way that allows for interactions between worker types and firm classes is key, since earnings may be non-monotonic in firm productivity and different workers rank identical firms differently. Our static model can accommodate both features.

Models with Markovian match effects and state dependence. In dynamic models workers often move based on the realization of the match effect ε_t, which is allowed to be serially correlated. Alternatively, ε_t may be thought of as a scalar human capital process. This is compatible with the assumptions of our dynamic model provided ε_t is first-order Markov, see Assumption 2. For example, in a wage posting model with match-specific heterogeneity workers may observe potential wages before deciding whether or not to move. While incompatible with Assumption 1, this is perfectly consistent with the dynamic model's assumptions provided mobility, the new firm's class, and the new wage are *jointly* first-order Markov.

To see this formally, consider an agent in period t with firm class k_t and wage Y_t. She draws an offer, (Y^*_{t+1}, k^*_{t+1}), jointly with a potential wage $\widetilde{Y}_{t+1}$ she would get should she decide not to move, all of which may depend on the current wage Y_t, firm class k_t, and type α. The decision to move is based on all this information. The realized firm class is then either $k_{t+1} = k_t$ with associated wage $\widetilde{Y}_{t+1}$, or $k_{t+1} = k^*_{t+1}$ with wage Y^*_{t+1}, depending on the outcome of the mobility decision. Assumption 2 is satisfied in this model, since the effective conditioning set is (α, Y_t, k_t).

Our dynamic model encompasses other mechanisms, such as endogenous search intensity along the lines of Bagger and Lentz (2014), where the previous wage may affect offers through an endogenous search decision. It also encompasses sequential contracting as in Postel-Vinay and Robin (2002), where the Bertrand competition is captured by the fact that the outside offer Y^*_{t+1} and the firm's wage counteroffer $\widetilde{Y}_{t+1}$ may depend on each other, and (α, Y_t, k_t) are sufficient statistics for the history. Related examples are contract posting models (as in Burdett and Coles, 2003, Shi, 2008), where the optimal contract is a tenure contract.

In the setting of Assumption 2 the wage conditional on moving depends on the past wage and the past firm. Our dynamic model allows for these selection effects. However, recovering underlying primitives such as distributions of wage offers Y^*_{t+1} would require making additional assumptions. In the absence of those, our framework allows one to identify the distributions of *realized* wages for job movers and stayers, as a function of worker and firm heterogeneity.

Time effects. Our static and dynamic models allow distributions to depend unrestrictedly on calendar time. Lise and Robin (2013) develop a model of sorting in a labor market with sequential contracting and aggregate shocks. Present values and earnings are functions of worker and firm heterogeneity, as well as of an aggregate state and the current bargaining position. Assumption 2 of our dynamic model is satisfied in this setting.

Outside our framework. However, non-Markovian earnings structures will violate the assumptions of our dynamic model. This will happen if the structural model allows for permanent-transitory earnings dynamics conditional on worker types, as in Hall and Mishkin (1982) for example. This will also happen in models that combine a sequential contracting mechanism (à la Postel-Vinay and Robin, 2002) with a match-specific effect. In this case agents need to keep track of both the match effect and the bargaining position, so the one-to-one mapping between earnings and the value to the worker no longer holds, making mobility decisions potentially dependent on the whole history of wages. Such environments are not nested in a framework such as ours, which only allows for uni-dimensional time-varying effects ε_t.

3 Identification

In this section we provide conditions for identification of earnings distributions for all worker types and firm classes, and worker type distributions for all firm classes, given two periods in the static model and four periods in the dynamic model. The analysis is conditional on a partition of firms into classes. In the next section we will show how to consistently estimate class membership $k(j)$, for each firm j.

3.1 Intuition in an interactive regression model

We first provide an intuition for identification of complementarities in a stationary specification of the interactive regression model of equation (1) with $T = 2$ periods, where we abstract from covariates. Consider job movers between two firms of classes k and $k' \neq k$, respectively, between periods 1 and 2. Here we study identification in a population where there is a continuum of workers moving from k to k'.[7] Log-earnings in each period are given by:

$$Y_{i1} = a(k) + b(k)\alpha_i + \varepsilon_{i1}, \quad Y_{i2} = a(k') + b(k')\alpha_i + \varepsilon_{i2}. \tag{3}$$

where $\mathbb{E}(\varepsilon_{it} \,|\, \alpha_i, k_{i1} = k, k_{i2} = k', m_{i1} = 1) = 0$. In this sample of job movers, the ratio $b(k')/b(k)$ is not identified without further assumptions.[8]

Consider now job movers from a firm in class k' to a firm in class k. Their log-earnings are given by:

$$Y_{i1} = a(k') + b(k')\alpha_i + \varepsilon_{i1}, \quad Y_{i2} = a(k) + b(k)\alpha_i + \varepsilon_{i2}.$$

By comparing differences in log-earnings in each class between these two subpopulations of job movers, we obtain:

$$\frac{b(k')}{b(k)} = \frac{\mathbb{E}_{kk'}(Y_{i2}) - \mathbb{E}_{k'k}(Y_{i1})}{\mathbb{E}_{kk'}(Y_{i1}) - \mathbb{E}_{k'k}(Y_{i2})}, \tag{4}$$

provided that the following condition holds:

$$\mathbb{E}_{kk'}(\alpha_i) \neq \mathbb{E}_{k'k}(\alpha_i), \tag{5}$$

where we have denoted $\mathbb{E}_{kk'}(Z_i) = \mathbb{E}(Z_i \,|\, k_{i1} = k, k_{i2} = k', m_{i1} = 1)$. This shows that, if (5) holds, then $b(k')/b(k)$ is identified from mean restrictions on job movers between k and k'. Conversely, if (5) does not hold then $b(k')/b(k)$ is not identified based on those restrictions. Note that (5) requires the types of workers moving from k to k' and from k' to k to differ. If $b(k') + b(k) \neq 0$, (5) is equivalent to:

$$\mathbb{E}_{kk'}\left(Y_{i1} + Y_{i2}\right) \neq \mathbb{E}_{k'k}\left(Y_{i1} + Y_{i2}\right), \tag{6}$$

so it can be empirically tested (under the maintained hypothesis of exogenous mobility).

[7]This intuitively means that this analysis will be relevant for data sets with a sufficient number of workers moving between firm classes. Our grouping of firms into classes is motivated by the incidental parameter bias due to low mobility. We will return to this issue in the estimation section.

[8]Model (3) is formally equivalent to a measurement error model where α_i is the error-free regressor and Y_{i2} is the error-ridden regressor. It is well-known that identification fails in general. For example, $b(k')/b(k)$ is not identified when ε_{i1}, ε_{i2}, and α_i are independent Gaussian random variables (Reiersøl, 1950).

An implication is that, when (5) does not hold, additivity of log-earnings in worker and firm attributes (that is, the $b(k)$'s being equal in all firms) is not testable based on mean restrictions. This analysis clarifies what can be learned from graphical illustrations of mean log-earnings before and after a job move event, which are often used to support additive specifications (e.g., Card et al., 2013). Strictly speaking, documenting symmetric wage gains and losses is not sufficient to demonstrate that wage functions are additive. As an example, in the theoretical model of Shimer and Smith (2000) wage gains and losses are symmetric around a job move, yet the wage function can feature any degree of complementarity between worker types and firm classes.[9]

A main goal of this paper is to establish that, by fully exploiting earnings information before and after a job move, complementarities can be identified and consistently estimated under a rank condition akin to (5). Such a condition will be satisfied quite generally. For example, it is satisfied in an extension of the model of Shimer and Smith (2000) with on-the-job search. In the supplementary appendix we evaluate the performance of our estimator to recover the contributions of worker and firm heterogeneity to earnings dispersion, when the data generating process follows this theoretical model. We show that our estimator recovers the wage functions and contributions of firms and workers to earnings dispersion, both under positive and negative assortative matching.

3.2 Identification with discrete worker types

In this subsection we consider the general static and dynamic models under Assumptions 1 and 2, respectively. We make no functional form assumptions on earnings distributions, except that we consider models where worker types α_i have finite support. Relying on discrete types is helpful for tractability, and we will use a finite mixture specification in our empirical implementation. However, in the supplementary appendix we also outline an extension to continuously distributed worker types. We start by presenting the main identifying equations.

3.2.1 Identifying equations

Static model on two periods. We first consider the static model on $T = 2$ periods, which suffice for identification. Let $F_{k\alpha}(y_1)$ denote the cumulative distribution function (cdf) of log-earnings in period 1, in firm class k, for worker type α. Let $F^m_{k'\alpha}(y_2)$ denote the cdf of log-

[9]In Figure S3 in the supplementary appendix we illustrate this point by simulating the Shimer Smith model under positive assortative matching, and showing the corresponding event study graph around job mobility.

earnings in period 2, for class k' and type α, for job movers between periods 1 and 2 (that is, when $m_{i1} = 1$). Let also $p_{kk'}(\alpha)$ denote the probability distribution of α_i for job movers between a firm of class k and another firm of class k'. Finally, let $q_k(\alpha)$ denote the distribution of α_i for workers in a firm of class k. All these distributions may be conditional on exogenous covariates X_{i1} and X_{i2}, although we omit the conditioning for conciseness.

The model imposes the following restrictions on the bivariate log-earnings distribution for job movers:

$$\Pr\left[Y_{i1}{\le}y_1, Y_{i2}{\le}y_2 \,|\, k_{i1}{=}k, k_{i2}{=}k', m_{i1}{=}1\right] = \int F_{k\alpha}(y_1)F^m_{k'\alpha}(y_2)p_{kk'}(\alpha)d\alpha. \tag{7}$$

To see why (7) holds, note that Y_{i1} is independent of (k_{i2}, m_{i1}) conditional on (α_i, k_{i1}). This is due to the fact that, by Assumption 1 (i), mobility is unaffected by log-earnings Y_{i1}, conditional on type and classes. Moreover, Y_{i2} is independent of (Y_{i1}, k_{i1}) conditional on $(\alpha_i, k_{i2}, m_{i1} = 1)$. This is due to the lack of dependence on the past after a job move in Assumption 1 (ii). In addition, we have the following decomposition of the cdf of log-earnings in period 1:

$$\Pr\left[Y_{i1} \le y_1 \,|\, k_{i1} = k\right] = \int F_{k\alpha}(y_1)q_k(\alpha)d\alpha. \tag{8}$$

The quantities in (7) and (8) allow documenting the sources of earnings inequality and the allocation of workers to firms. For example, the $F_{k\alpha}$ are informative about the presence of complementarities in the earnings function. Differences of $q_k(\alpha)$ across k are indicative of cross-sectional sorting. Moreover, from the $q_k(\alpha)$, $p_{kk'}(\alpha)$, and data on transitions between classes, one can recover estimates of type-specific transition probabilities between classes, which are informative about dynamic sorting patterns. We will report estimates of all these quantities in the empirical analysis.

Dynamic model on four periods. In the dynamic model on $T = 4$ periods, let $G^f_{y_2,k\alpha}(y_1)$ (for "forward") denote the cdf of log-earnings in period 1, in a firm class k, for a worker of type α who does not change firm between periods 1 and 2 and earns y_2 in period 2. Let $G^b_{y_3,k'\alpha}(y_4)$ (for "backward") be the cdf of Y_{i4}, in firm class k', for a worker of type α who does not change firm between periods 3 and 4 and earns y_3 in period 3. Lastly, let $p_{y_2y_3,kk'}(\alpha)$ denote the type distribution of workers who stay in the same firm of class k between periods 1 and 2, move to another firm of class k' in period 3, remain in that firm in period 4, and earn y_2 and y_3 in periods 2 and 3, respectively. For conciseness we again omit the conditioning on covariates.

The bivariate cdf of log-earnings Y_{i1} and Y_{i4} is, for workers who change firm between periods 2 and 3:

$$\Pr\left[Y_{i1} \leq y_1, Y_{i4} \leq y_4 \,|\, Y_{i2}{=}y_2, Y_{i3}{=}y_3, k_{i1}{=}k_{i2}{=}k, k_{i3}{=}k_{i4}{=}k', m_{i1}{=}0, m_{i2}{=}1, m_{i3}{=}0\right]$$
$$= \int G^f_{y_2,k\alpha}(y_1) G^b_{y_3,k'\alpha}(y_4) p_{y_2y_3,kk'}(\alpha) d\alpha. \tag{9}$$

Equation (9) is a consequence of Assumption 2, which is a first-order Markov assumption on the process $(Y_{it}, k_{it}, m_{i,t-1})$, where in addition m_{it} can only depend on Y_{it} and k_{it} but not on $m_{i,t-1}$. In particular, by Assumption 2 (ii), Y_{i4} is independent of past mobility, firm classes, and earnings, conditional on $(\alpha_i, Y_{i3}, k_{i4}, k_{i3}, m_{i3})$. Similarly, Y_{i1} can be shown to be independent of future classes, earnings and mobility conditional on $(\alpha_i, Y_{i2}, k_{i1}, k_{i2}, m_{i1})$.

In addition, here $F_{k\alpha}$ denotes the cdf of log-earnings Y_{i2} for workers in firm class k who remain in the same firm in periods 1 and 2 (that is, $m_{i1} = 0$), while $q_k(\alpha)$ denotes the distribution of α_i for these workers. The joint cdf of log-earnings in periods 1 and 2 is:

$$\Pr\left[Y_{i1} \leq y_1, Y_{i2} \leq y_2 \,|\, k_{i1} = k_{i2} = k, m_{i1} = 0\right] = \int G^f_{y_2,k\alpha}(y_1) F_{k\alpha}(y_2) q_k(\alpha) d\alpha. \tag{10}$$

The mathematical structure of (9)-(10) is analogous to that of (7)-(8). This is useful to analyze the static and dynamic models using similar methods. Intuitively, the conditioning on log-earnings Y_{i2} and Y_{i3} immediately before and after the job move ensures conditional independence of log-earnings Y_{i1} and Y_{i4}, even though in this model earnings have a direct effect on job mobility and respond dynamically to lagged earnings and previous firm classes.

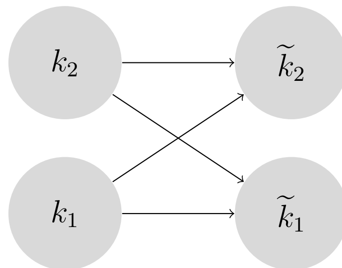
3.2.2 Identification analysis

We start by considering the static model on two periods. Since the dynamic model has a similar mathematical structure the identification arguments will be closely related. Let L be the number of points of support of worker types, and let us denote the types as $\alpha_i \in \{1, ..., L\}$. We assume that L is known. In the application we will check sensitivity by varying L. All distributions below may be conditional on (X_{i1}, X_{i2}), although we omit the conditioning for conciseness.

In this finite mixture model, (7) and (8) imply restrictions on the cdfs $F_{k\alpha}$ and $F^m_{k'\alpha}$, and on the probabilities $p_{kk'}(\alpha)$ and $q_k(\alpha)$. We now provide conditions under which these quantities are identified. We start with a definition.

Definition 1. *An alternating cycle of length R is a pair of sequences of firm classes $(k_1, ..., k_R)$ and $(\widetilde{k}_1, ..., \widetilde{k}_R)$, with $k_{R+1} = k_1$, such that $p_{k_r,\widetilde{k}_r}(\alpha) \neq 0$ and $p_{k_{r+1},\widetilde{k}_r}(\alpha) \neq 0$ for all r in $\{1, ..., R\}$ and α in $\{1, ..., L\}$.*

Figure 1: An alternating cycle of length $R = 2$



Assumption 3. *(mixture model, static)*
(i) For any two firm classes $k \neq k'$ in $\{1, ..., K\}$, there exists an alternating cycle $(k_1, ..., k_R)$, $(\widetilde{k}_1, ..., \widetilde{k}_R)$, such that $k_1 = k$ and $k_r = k'$ for some r, and such that the scalars $a(1), ..., a(L)$ are all distinct, where:

$$a(\alpha) = \frac{p_{k_1,\widetilde{k}_1}(\alpha) p_{k_2,\widetilde{k}_2}(\alpha) ... p_{k_R,\widetilde{k}_R}(\alpha)}{p_{k_2,\widetilde{k}_1}(\alpha) p_{k_3,\widetilde{k}_2}(\alpha) ... p_{k_1,\widetilde{k}_R}(\alpha)}.$$

In addition, for all k, k', possibly equal, there exists an alternating cycle $(k'_1, ..., k'_R)$, $(\widetilde{k}'_1, ..., \widetilde{k}'_R)$, such that $k'_1 = k$ and $\widetilde{k}'_r = k'$ for some r.

(ii) There exist finite sets of M values for y_1 and y_2 such that, for all r in $\{1, ..., R\}$, the matrices $A(k_r, \widetilde{k}_r)$ and $A(k_r, \widetilde{k}_{r+1})$ have rank L, where:

$$A(k, k') = \left\{ \Pr \left[Y_{i1} \leq y_1, Y_{i2} \leq y_2 \,|\, k_{i1} = k, k_{i2} = k', m_{i1} = 1 \right] \right\}_{(y_1, y_2)}.$$

Assumption 3 requires that any two firm classes k and k' belong to an alternating cycle. An example is given in Figure 1, in which case the presence of an alternating cycle requires that there is a positive proportion of every worker type in the sets of movers from k_1 to $\widetilde{k}_1$, k_1 to $\widetilde{k}_2$, k_2 to $\widetilde{k}_1$, and k_2 to $\widetilde{k}_2$, respectively. Existence of cycles implies graph connectedness, in the sense of AKM (Abowd et al., 2002). At the same time, connectedness here is at the firm class level. A specific feature of our fully nonlinear setting is the need for every firm class to contain job movers of all types of workers. This may be demanding empirically, and the condition may fail in some models of matching. The requirement on cycles can be relaxed, at the cost of loosing point-identification of some of the quantities of interest.[10] Alternatively, one may impose more structure, as in the interactive regression model (1).

[10]In the supplementary appendix we illustrate this in a model where worker types and firm classes are ordered, there is strong positive assortative matching, and workers only move between adjacent firm classes.

Assumption 3 (i) requires some asymmetry in worker type composition between different firm classes. This condition requires non-random mobility, as it fails when $p_{kk'}(\alpha)$ does not depend on (k, k'). Also, part (i) fails when $p_{kk'}(\alpha)$ is symmetric in (k, k'). This situation arises in the model of Shimer and Smith (2000) in the absence of on-the-job search, as we discuss in the supplementary appendix. In the mixture model analyzed here, the presence of asymmetric job movements between firm classes is crucial for identification. This is similar to the case of the simple interactive regression model studied above, see (6).

Finally, Assumption 3 (ii) is a rank condition. It will be satisfied if, in addition to part i), for all r the distributions $F_{k_r,1}$, ..., $F_{k_r,L}$ are linearly independent, and similarly for $F_{\widetilde{k}_r,1}$, ..., $F_{\widetilde{k}_r,L}$, $F^m_{k_r,1}$, ..., $F^m_{k_r,L}$, and $F^m_{\widetilde{k}_r,1}$, ..., $F^m_{\widetilde{k}_r,L}$.

The next result shows that, with only two periods and given the structure of the static model, both the type-and-class-specific earnings distributions and the proportions of worker types can be uniquely recovered. The intuition for the result is similar to that in the simple interactive regression model above. Due to the discrete heterogeneity setting, identification is up to an arbitrary choice of labeling of the latent worker types. The proof is in Appendix A.

Theorem 1. *Let $T = 2$, and let Assumptions 1 and 3 hold. Suppose that firm classes are observed. Then, up to labeling of the types α, $F_{k\alpha}$ and $F^m_{k'\alpha}$ are identified for all (α, k, k'). Moreover, for all pairs (k, k') for which there are job moves from k to k', $p_{kk'}(\alpha)$ is identified for all α, for the same labeling. Lastly, the type proportions $q_k(\alpha)$ in the first period are all identified for the same labeling.*

Dynamic model. A similar approach allows us to establish identification of the dynamic mixture model with discrete worker heterogeneity on four periods under Assumption 2. Exploiting the link between (7) and (9) on the one hand, and (8) and (10) on the other hand, we obtain identification as a corollary to Theorem 1, see Corollary S1 in the supplementary appendix. The required assumptions, particularly on the existence of cycles, are stronger than in the static case.

4 Estimation

In the previous section we have provided conditions under which earnings distributions are identified in the presence of sorting and complementarities. These results hold at the firm class level k_{it}, where in principle the k_{it} could coincide with the firm j_{it}. However, in matched employer employee panel data sets of typical sizes, estimating models with complementarities, dynamics, and two-sided heterogeneity may be ill-behaved due to the incidental parameter biases caused by the large number of firm-specific parameters that are solely identified from job movements. For this reason, we use a dimension reduction method to partition firms into classes. We now describe a computationally tractable two-step grouped fixed-effects approach, where we classify firms in a first step and estimate earnings and mobility parameters in a second step.

4.1 Recovering firm classes using k-means clustering

In both the static and dynamic models described in Section 2, the distributions of log-earnings Y_{it} and characteristics X_{it}, and the probabilities of mobility m_{it}, are all allowed to depend on firm classes k, but not on the identity of the firm within class k. In other words, unobservable firm heterogeneity operates at the level of firm classes in the model, not at the level of individual firms. For example, in (8) the first period's distribution of log-earnings in firm j does not depend on j beyond its dependence on firm class $k = k(j)$:

$$\Pr\left[Y_{i1} \leq y_1 \,|\, j_{i1} = j\right] \,=\, \int F_{k\alpha}(y_1) q_k(\alpha) d\alpha, \tag{11}$$

where the left-hand side thus only depends on $k = k(j)$. This observation motivates classifying firms into classes in terms of their earnings distributions, as we now explain.

We propose partitioning the J firms in the sample into classes by solving the following weighted k-means problem:

$$\min_{k(1),...,k(J),H_1,...,H_K} \sum_{j=1}^{J} n_j \int \left(\widehat{F}_j(y) - H_{k(j)}\left(y\right)\right)^2 d\mu(y), \tag{12}$$

where $\widehat{F}_j$ denotes the empirical cdf of log-earnings in firm j, n_j is the number of workers in firm j, μ is a discrete or continuous measure, $k(1), ..., k(J)$ denotes a partition of firms into K classes, and $H_1, ..., H_K$ are cdfs. We minimize (12) with respect to all possible partitions and to class-specific cdfs. While global minima in k-means may be challenging to compute, k-means algorithms are widely used in many fields and efficient heuristic computational methods have been developed (e.g., Steinley, 2006).

Through the classification in (12) we estimate firm classes as "discrete fixed-effects", allowing them to be correlated to firm-specific covariates. In our application on short panels we will assume that the firms' classification is time-invariant, and we will correlate the estimated classes *ex-post* to firm observables.

To provide a formal justification for the classification, in the supplementary appendix we consider a setting where the model (either static or dynamic) is well-specified and there exists a partition of firms into K classes in the population. We consider an asymptotic sequence where both the number of firms and the number of workers per firm tend to infinity. Using a result from Bonhomme and Manresa (2015) we show that estimated firm classes, $\widehat{k}(j)$, converge uniformly to the population ones up to an arbitrary labeling as the sample size grows. As a result, the asymptotic distribution of parameter estimates in the second step is not affected by the estimation of firm classes.

Two remarks are in order. First, the labeling of the classes is arbitrary. This labeling does not affect variance decomposition exercises. However, a structural interpretation of the firm classes in terms of productivity would require additional assumptions. Eeckhout and Kircher (2011) show that it can even be impossible to recover such a productivity ordering from wage data only.

Second, the classification fails to be identified when two firm classes have identical earnings distributions in the cross-section. This can happen if one firm offers a higher earnings schedule but has lower-type workers, and another one offers a lower earnings schedule but has higher-type workers. In some environments without firm capacity constraints, such as Postel-Vinay and Robin (2002), the upper bound of earnings in the firm is increasing in firm productivity so firm-specific distributions are all different and firms may be consistently classified based on their earnings distributions. It is difficult to obtain similar guarantees in models with capacity constraints. Nevertheless, note that identification can survive even when some firms have identical mean earnings, provided the *distributions* of earnings differ, as shown by our parameterization under negative assortative matching of the Shimer and Smith (2000) model in the supplementary appendix.

In the empirical analysis we attempt to bring additional information beyond cross-sectional earnings to learn about firm heterogeneity, in several ways. Going beyond wage information has been advocated in a the structural literature (e.g., Eeckhout and Kircher, 2011, Bagger and Lentz, 2014, Hagedorn et al., 2017, Bartolucci et al., 2015). As robustness checks, we use re-classification and random-effects methods, which rely on longitudinal information on both earnings and mobility. We also experiment, albeit without a structural model, with bringing other information on the firm in order to inform the classification, such as firm value added and measures of worker flows.

4.2 Two-step grouped fixed-effects estimation

Our two-step grouped fixed-effects estimation strategy is as follows. In the first step (classification) we estimate the firm classes $\widehat{k}(j)$ for all firms j in the sample, by solving a classification problem such as (12). In the second step (estimation) we impute a class $\widehat{k}_{it} = \widehat{k}(j_{it})$ to each worker-period observation in the sample, and we estimate the model conditional on the $\widehat{k}_{it}$'s.

To describe the estimation step we consider a specification where workers belong to L latent types, and the model is parametric given worker and firm heterogeneity. Let us first focus on a two-period version of the static model and a four-period version of the dynamic model, both of which we will estimate on Swedish data. In the static case we let $f_{k\alpha}(y;\theta_f)$ (first-period earnings), $f^m_{k\alpha}(y;\theta_{f^m})$ (second-period earnings for job movers), $q_k(\alpha;\theta_q)$ (worker-type proportions), and $p_{kk'}(\alpha;\theta_p)$ (worker-type proportions for job movers) be indexed by parameter vectors $\theta_f, \theta_{f^m}, \theta_q, \theta_p$. In our baseline specification we will let both earnings densities be log-normal with (k,α)-specific means and variances.[11] That is, means and variances of log-earnings are allowed to differ between all combinations of worker types and firm classes. In addition, in the time dimension we will allow for full interactions between firm classes and time indicators, as well as unrestricted non-stationary variances. Lastly, we will treat all $q_k(\alpha)$ and $p_{kk'}(\alpha)$ as unrestricted parameters.

Following the spirit of the identification strategy, we first estimate log-earnings densities using job movers only, and we then estimate worker type proportions in the first period using both job movers and job stayers.[12] Under the assumption that worker types and earnings realizations are independent across workers conditional on mobility indicators and firm classes, the log-likelihood of job movers conditional on mobility patterns and estimated firm classes takes the following form (N_m denoting the number of job movers):

$$\sum_{i=1}^{N_m}\sum_{k=1}^{K}\sum_{k'=1}^{K}\mathbf{1}\{\widehat{k}_{i1}=k\}\mathbf{1}\{\widehat{k}_{i2}=k'\}\ln\left(\sum_{\alpha=1}^{L}p_{kk'}(\alpha;\theta_p)f_{k\alpha}(Y_{i1};\theta_f)f^m_{k'\alpha}(Y_{i2};\theta_{f^m})\right). \tag{13}$$

In turn, the log-likelihood of all workers in period 1 is:

$$\sum_{i=1}^{N}\sum_{k=1}^{K}\mathbf{1}\{\widehat{k}_{i1}=k\}\ln\left(\sum_{\alpha=1}^{L}q_k(\alpha;\theta_q)f_{k\alpha}(Y_{i1};\widehat{\theta}_f)\right). \tag{14}$$

[11]We also estimated a specification based on a mixture of normals, which gave results similar to the baseline specification (see the supplementary appendix).

[12]Proceeding in this way has the advantage of recovering earnings parameters from job movements directly, albeit at some efficiency cost. In practice we estimate the type proportions of job stayers in the last step, and combine them with the $p_{kk'}(\alpha)$ to recover the unconditional proportions $q_k(\alpha)$.

Hence, conditional on the estimated firm classes, (13) and (14) are standard single-agent correlated random-effects log-likelihood functions. We estimate $\widehat{\theta}_f, \widehat{\theta}_{f^m}, \widehat{\theta}_p$ by maximizing (13), and then $\widehat{\theta}_q$ by maximizing (14). We use the EM algorithm (Dempster et al., 1977) for computation.

Exogenous worker covariates can be readily incorporated in estimation, by modifying the form of the likelihood. In the empirical analysis, given the short length of the panel we will use a nonstationary specification, and relate the latent worker types to time-invariant covariates *a posteriori*. This will allow us to account for sorting on observables such as education or cohort. In the supplementary appendix we explain how we modify (14) for this purpose.

Dynamic model. We use a similar approach for the dynamic finite mixture model on four periods, see equations (9) and (10). In this case we specify the conditional mean of Y_{i4} given Y_{i3} and worker and firm heterogeneity as $\mu_{4k'\alpha} + \rho_{4|3}Y_{i3}$, where $\mu_{4k'\alpha}$ is a (k', α)-specific intercept. Likewise, the conditional mean of Y_{i1} given Y_{i2} and worker and firm heterogeneity is $\mu_{1k\alpha} + \rho_{1|2}Y_{i2}$. The parameters $\rho_{4|3}$ and $\rho_{1|2}$ capture the persistence of log-earnings within job. For parsimony we have imposed that those parameters are homogeneous across worker types and firm classes, although this could easily be relaxed with a larger sample.

In addition we specify the mean of (Y_{i2}, Y_{i3}) for job movers between classes k and k' as $(\mu_{2k\alpha} + \xi_2(k'), \mu_{3k'\alpha} + \xi_3(k))$. The term $\xi_2(k')$ reflects that, conditional on moving between k and k', mean log-earnings before the move can differ with the firm of destination, due to the presence of *endogenous mobility*. The term $\xi_3(k)$ reflects that the previous firm is allowed to have a direct effect on log-earnings after a move, through the presence of *state dependence*. Neither of those effects is allowed for in the static version of the model. We specify the mean of (Y_{i2}, Y_{i3}) for job stayers in a firm of class k as $(\mu^s_{2k\alpha}, \mu^s_{3k\alpha})$. Lastly, for both stayers and movers we let the covariance matrices vary with firm classes.[13]

Given estimates $\widehat{\rho}_{4|3}$ and $\widehat{\rho}_{1|2}$ of the persistence parameters, the other parameters can be estimated using a very similar approach as in the static case, based on the following log-likelihood functions:

$$\sum_{i=1}^{N_m} \sum_{k=1}^{K} \sum_{k'=1}^{K} \mathbf{1}\{\widehat{k}_{i2} = k\}\mathbf{1}\{\widehat{k}_{i3} = k'\} \times ...$$

$$\ln\left(\sum_{\alpha=1}^{L} p_{kk'}(\alpha; \theta_p) f^f_{Y_{i2},k\alpha}(Y_{i1}; \widehat{\rho}_{1|2}, \theta_{f^f}) f^m_{kk'\alpha}(Y_{i2}, Y_{i3}; \theta_{f^m}) f^b_{Y_{i3},k'\alpha}(Y_{i4}; \widehat{\rho}_{4|3}, \theta_{f^b})\right), \quad (15)$$

[13] We impose that the best linear predictors in the regressions of Y_{i3} on Y_{i2}, for both stayers (denoted as $\rho^s_{3|2}$) and movers ($\rho^m_{3|2}$), do not depend on worker types or firm classes, and that the residual variances in the case of movers only depend on k'. This could be relaxed with a large enough sample.

and:

$$\sum_{i=1}^{N}\sum_{k=1}^{K}\mathbf{1}\{\widehat{k}_{i2}=k\}\ln\left(\sum_{\alpha=1}^{L}q_k(\alpha;\theta_q)f^f_{Y_{i2}k\alpha}(Y_{i1};\widehat{\rho}_{1|2},\widehat{\theta}_{ff})f^s_{k\alpha}(Y_{i2},Y_{i3};\theta_{f^s})f^b_{Y_{i3},k'\alpha}(Y_{i4};\widehat{\rho}_{4|3},\widehat{\theta}_{f^b})\right).\quad(16)$$

We estimate $\widehat{\theta}_p, \widehat{\theta}_{ff}, \widehat{\theta}_{f^m}, \widehat{\theta}_{f^b}$ based on (15), and then $\widehat{\theta}_q, \widehat{\theta}_{f^s}$ based on (16), using the EM algorithm in both cases.

While it is in principle possible to estimate $\rho_{4|3}$ and $\rho_{1|2}$ by maximizing a joint likelihood function across movers and stayers with respect to all parameters, doing so would be computationally cumbersome. A convenient alternative, which we adopt in the empirical analysis, is to estimate these parameters in an initial step based on covariance restrictions. Under the assumption that the effect of worker types on mean log-earnings is constant over time within firm, simple restrictions on the ρ's can be obtained by exploiting the particular form of the conditional means of Y_{i4} given Y_{i3} and Y_{i1} given Y_{i2}, respectively. We provide details on the covariance-based estimation of $\rho_{4|3}$ and $\rho_{1|2}$ in the supplementary appendix.

The estimation approach outlined in this section can be modified in several ways that we will implement empirically. A first extension, described in the supplementary appendix, is a model-based re-classification. Given estimates of the θ parameters one may re-classify every firm j to the class $k = \widetilde{k}(j)$ which corresponds to the maximal value of the k-specific likelihoods of firm j's observations. This approach can be iterated further. In addition, while we have described estimation in the context of finite mixture models the two-step approach can be used in other settings. Relevant examples are regression models such as the AKM model and its interactive counterparts (1) and (2) that allow for complementarities or dynamics. In such models two-step grouped fixed-effects methods deliver computationally convenient estimation algorithms based on mean and covariance restrictions, as we show in detail in the supplementary appendix.

5 Empirical results I: Static model

We now present results for the static model on the Swedish data. Here we report estimates of firm classes, worker types and earnings based on our preferred specification. We provide evidence of the robustness of our findings in the supplementary appendix.

Data. We use administrative data covering the entire working age population in Sweden between 1997 and 2008. We follow Friedrich et al. (2014) for sample selection and construction of monthly log-earnings. We estimate the static model on males working in the private sector in 2002 and 2004. We keep workers who are both fully employed in the same firm in 2002 and fully

Table 1: Data description, by estimated firm classes

class:	1	2	3	4	5	6	7	8	9	10	all
number of workers	16,868	50,906	74,073	76,616	80,562	66,120	105,485	61,272	47,164	20,709	599,775
number of firms	5,808	6,832	4,983	5,835	3,507	4,149	3,672	3,467	2,886	2,687	43,826
mean firm reported size	12.43	20.92	42.68	28.47	65.06	32.30	60.08	51.24	54.16	50.86	37.59
number of firms $\geq$ 10 (actual size)	160	1,034	1,519	1,357	1,192	930	999	855	632	415	9,093
number of firms $\geq$ 50 (actual size)	7	87	260	225	270	162	245	183	147	52	1,638
% high school drop out	28.5%	27.8%	25.9%	26.8%	22.2%	23.8%	18.9%	12.9%	6.1%	3.2%	20.6%
% high school graduates	61.3%	63.4%	62.3%	63.3%	59.1%	62.7%	58.4%	49.3%	34.9%	25.6%	56.7%
% some college	10.2%	8.8%	11.8%	9.9%	18.7%	13.5%	22.8%	37.8%	59.0%	71.2%	22.7%
% workers younger than 30	24.3%	19.5%	19.8%	17.5%	18.6%	15.4%	13.8%	14.3%	15.0%	14.3%	16.8%
% workers between 31 and 50	54.1%	54.6%	55.0%	56.2%	56.0%	57.6%	58.5%	58.9%	60.0%	64.2%	57.2%
% workers older than 51	21.7%	25.9%	25.1%	26.3%	25.5%	27.0%	27.6%	26.8%	25.0%	21.5%	26.0%
% workers in manufacturing	24.3%	39.3%	46.8%	53.0%	51.5%	52.0%	53.0%	40.3%	31.5%	7.6%	45.4%
% workers in services	39.3%	32.1%	23.3%	19.7%	14.4%	15.0%	16.0%	29.7%	52.1%	72.6%	25.3%
% workers in retail and trade	26.4%	19.0%	24.9%	10.6%	29.3%	7.9%	8.4%	17.7%	14.8%	18.7%	16.7%
% workers in construction	9.9%	9.6%	5.1%	16.8%	4.9%	25.1%	22.5%	12.3%	1.5%	1.1%	12.6%
mean log-earnings	9.69	9.92	10.01	10.06	10.15	10.16	10.24	10.36	10.50	10.77	10.18
variance of log-earnings	0.101	0.054	0.085	0.051	0.102	0.051	0.077	0.096	0.109	0.173	0.124
skewness of log-earnings	-1.392	-0.709	0.345	0.019	0.576	0.433	0.474	0.703	0.385	1.001	0.582
kurtosis of log-earnings	7.780	14.093	9.017	15.565	7.788	14.763	10.033	8.141	6.651	6.984	7.400
between-firm variance of log-earnings	0.0462	0.0044	0.0036	0.0018	0.0032	0.0016	0.0016	0.0045	0.0057	0.0435	0.0475
mean log-value-added per worker	14.48	14.97	15.54	15.21	15.82	15.26	15.61	15.69	15.76	15.78	15.30

Notes: Males, fully employed in the same firm 2002 and 2004, continuously existing firms. Actual size is the number of workers per firm in our sample. Figures for 2002.

employed in the same firm in 2004, and firms with at least one fully-employed worker during the period. In Appendix B we provide details on the Swedish context and sample construction. We define job movements in a conservative way, which we describe in detail in the appendix. This results in low mobility rates, with a proportion of job movers to job stayers of 3.3% in the sample.

Firm classes. As described in Section 4, we estimate firm classes using a weighted k-means algorithm with $10,000$ randomly generated starting values. We use firms' cdfs of 2002 log-earnings on a grid of 20 percentiles of the overall log-earnings distribution. We weight measurements by firm size, and only include job stayers in the classification.

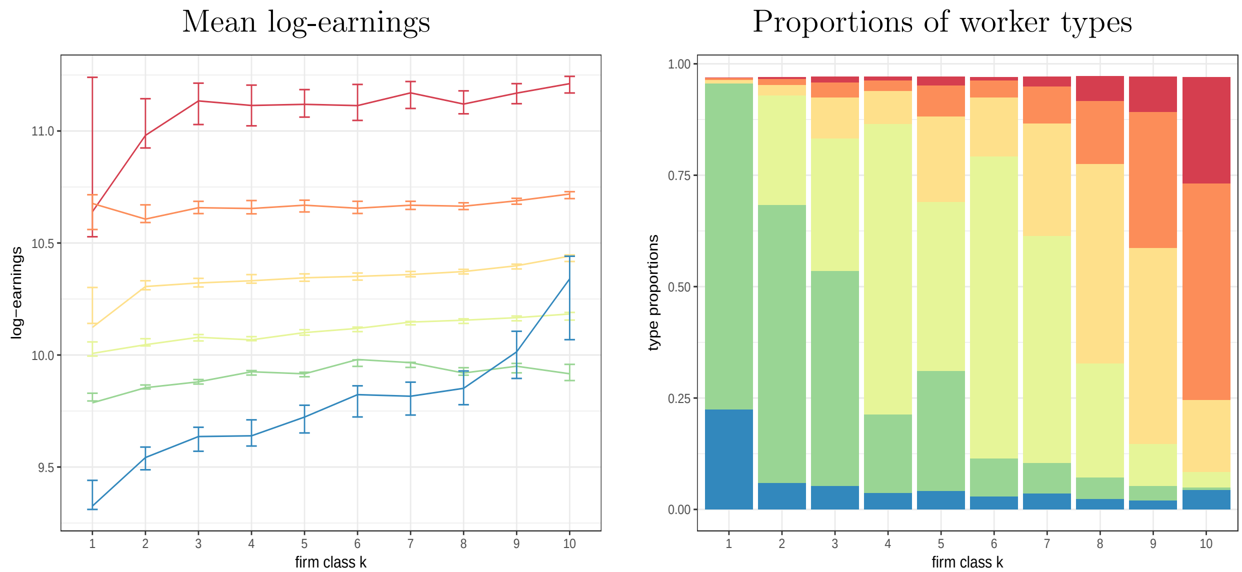
Table 1 provides summary statistics on the estimated firm classes for our baseline choice of number of classes $K = 10$. We order firm classes according to mean log-earnings in each class. Classes capture substantial heterogeneity between firms. The between-firm-class log-earnings variance is 0.0421, that is, 89% of the overall between-firm variance. This suggests that assuming homogeneity within each of the 10 classes might not result in major losses of information, at least in terms of variance of log-earnings. Classes differ also in terms of second, third, and fourth-order log-earnings moments. In addition there are substantial differences between classes in terms of worker characteristics. While lower classes (according to their mean log-earnings) show high percentages of high school dropouts and low percentages of workers with some college, higher classes show the opposite pattern. Lower classes also tend to have higher percentages of workers less than 30 years old, and lower percentages of workers between 30 and 50, while higher classes have more workers between 30 and 50. This relationship broadly reflects the life cycle pattern of earnings in these data.

Firm size reported by the firm tends to increase with firm class, although the relationship is not monotonic. Classes 1 and 2 contain smaller firms than the other classes. There is also evidence of both between- and within-sector variation between classes, which is not monotonic in mean earnings. For example, the proportion of workers in services is U-shaped in firm class, and that in manufacturing is inverse U-shaped. Lastly, log value added per worker tends to increase with firm class, although again there is not a monotonic relationship. Moreover, classes explain only 13.2% of the between-firm variance in log value added per worker.

In the supplementary appendix we describe some patterns of mobility and earnings across firm classes. In Table S3 we report the number of movers between all pairs of classes. There is substantial worker mobility between firm classes, especially between adjacent classes. This is important since our identification strategy is based on exploiting mobility. In Figure S6 we show means of log-earnings for workers moving between different firm classes k and k'. The graph shows that moves in either direction (i.e., moving "up" or "down") tend to be associated with different earnings levels. We emphasized the importance of such asymmetric patterns in our identification analysis, see equation (6).

Wages, worker heterogeneity and firm heterogeneity. Our baseline estimates are based on a Gaussian finite mixture model with $L = 6$ types of workers and $K = 10$ firm classes. As explained in Section 4 we estimate earnings distributions on the sample of job movers between 2002-2004, as well as the type proportions of movers. We then estimate proportions of worker types of job stayers in 2002. Maximum likelihood estimation of finite mixture models is often subject to local maxima, and our setting is no exception. In addition, in some of the locally optimal solutions some worker types only move within a subset of firm classes, resulting in

Figure 2: Main parameter estimates of the static model



Notes: Static model, 2002-2004. The left graph plots estimates of the means of log-earnings distributions, by worker type and firm class. The $K = 10$ firm classes (on the x-axis) are ordered by mean log-earnings. On the y-axis we report estimates of mean log-earnings for $L = 6$ worker types. The right graph shows estimates of the proportions of worker types in each firm class. Left: brackets indicate parametric bootstrap 2.5% and 97.5% quantiles (200 replications).

unstable parameter estimates. In the supplementary appendix we describe how we use the EM algorithm to explore the likelihood function. We also explain how we use a measure of network connectedness recently studied in Jochmans and Weidner (2017) to select our preferred estimates.

On the left panel of Figure 2 we plot estimates of the means of log-earnings for each firm class and each worker type. On the x-axis, firm classes are ordered by mean log-earnings. The brackets show 95% confidence intervals based on the parametric bootstrap.[14] The results show clear evidence of worker heterogeneity. They also show some variation in log-earnings between firm classes, although to a lesser extent. Moreover, lower-type workers (where "lower" and "higher" types refer to low and high mean log-earnings) appear to gain the most from working in a higher-wage firm. This suggests the presence of some complementarity between firms and

[14]The bootstrap draws are conditional on worker and firm links in the data, and firm classes are re-estimated in each replication. This bootstrap procedure provides a measure of parameter uncertainty that accounts for uncertainty in firm classes. In the supplementary appendix we derive the asymptotic distribution of the estimators, and we provide implementation details.

Table 2: Variance decomposition and reallocation exercise in the static model

Variance decomposition $(\times 100)$				
$\frac{Var(\alpha)}{Var(y)}$	$\frac{Var(\psi)}{Var(y)}$	$\frac{2Cov(\alpha,\psi)}{Var(y)}$	$\frac{Var(\varepsilon)}{Var(y)}$	$Corr(\alpha,\psi)$
60.03	2.56	12.17	25.24	49.13
(0.85)	(0.16)	(0.39)	(0.59)	(0.86)
Reallocation exercise $(\times 100)$				
Mean	Median	10%-quantile	90%-quantile	Variance
0.50	0.58	2.60	-1.24	-1.12
(0.10)	(0.11)	(0.19)	(0.31)	(0.11)

Notes: Static model, 2002-2004. Top panel: α is the worker effect, ψ is the firm effect, variance decomposition based on a linear regression of simulated 2002 log-earnings. Bottom panel: differences in means, quantiles and variances of log-earnings between a sample where workers are randomly reallocated to firms and the original sample. See the supplementary appendix for details on the computation. $1,000,000$ *simulations. Parametric bootstrap standard errors in parentheses (*200 *replications).*

lower-type workers, which we will further explore below.

On the right panel of Figure 2 we report the estimated proportions of worker types in each firm class. The results show how the composition of worker types differs markedly across firm classes. For example, the lowest-class firms (in terms of mean log-earnings) employ mostly the bottom two worker types, while the highest-class firms employ mostly the top three worker types. Overall, the two graphs in Figure 2 suggest that variation in log-earnings between firm classes is mainly due to firms employing different workers, rather than differences in earnings for a given worker type.

Variance decomposition and reallocation. We next report the results of several exercises that illustrate how earnings and heterogeneity relate to each other. We start with a decomposition of the variance of 2002 log-earnings. In the literature since Abowd et al. (1999) it is common to decompose the variance of log-earnings, net of observed covariates, into four components: the variance of worker effects α (that is, coefficients of worker type indicators), the variance of firm effects ψ (i.e, coefficients of firm class indicators), twice the covariance between the two, and the variance of residuals ε. In our nonlinear model a similar decomposition can be performed by working with a linear projection of log-earnings on worker type indicators and firm class indicators, in a regression without interactions. The results of the decomposition reported on the top panel of Table 2 show two main features. First, worker heterogeneity explains substantially more variation in earnings than firm heterogeneity. Differences in firm classes only account for 2.6% of the variance, compared to 60% for the part due to differences in worker types. The second main finding is that the part explained by the covariance is substantial. The correlation between worker and firm effects is 49%, which suggests the presence of strong sorting between workers and firms. This is in line with the evidence documented on the right panel of Figure 2.

As a first way to quantify the economic magnitude of complementarities, we next assess the explanatory power of worker types and firm classes when entered interactively as opposed to additively in the regression. The R^2 coefficient in the linear regression is 74.8%, while in the regression that includes all interactions between worker type indicators and firm class indicators the R^2 is 75.8%. This suggests that, while the left panel of Figure 2 shows the presence of some complementarity between firms and lower-type workers, those complementarities explain only a small part of the overall variance of log-earnings.

We next consider the impact on log-earnings of a reallocation exercise where workers are randomly allocated to firms. Such an exercise aims at assessing the contribution of sorting to the distribution of earnings. We assume that earnings functions, for all worker types and firm classes, are not affected by the reallocation, hence abstracting from equilibrium effects. We show the results of the reallocation on the bottom panel of Table 2. In the first column we report the estimate of the difference in mean log-earnings between a counterfactual sample where workers are randomly allocated among firms and our sample, using our estimates that account for the presence of complementarities. In an additively separable economy between workers and firms, such as under the AKM model, there should be no effect of the reallocation on mean log-earnings (e.g., Graham et al., 2014). However we find a positive mean impact (.5%), which suggests that the effect of complementarities on average log-earnings is statistically significant but quantitatively small. The positive sign is in line with the fact that the mean worker type tends to increase with firm class k, while complementarities are somewhat stronger in low firm classes.[15]

Moreover, in our distributional framework we are able to estimate the earnings entire distribution corresponding to a given reallocation of workers to firms. In columns 2 to 4 of the bottom panel of Table 2, we show the differences in medians and 10% and 90% percentiles of log-earnings between the random allocation and our sample. We also report differences in variances in the last column. We see that, while the median effect is in line with the mean

[15]To provide an intuition, consider the regression model (1). In this specification the difference between mean outcomes in a population where workers are randomly allocated to firms and in our data is, abstracting from time indices for clarity: $\mathbb{E}^{random}(Y_i) - \mathbb{E}(Y_i) = -\operatorname{Cov}(b(k_i), \mathbb{E}(\alpha_i \mid k_i))$.

effect, the bottom of the distribution would tend to benefit in the random allocation, while the top would be hurt. Those differences reflect both the fact that log-earnings are less dispersed in the random allocation, as shown by the reduction in variance, as well as the presence of complementarities at the bottom of the distribution.

Robustness. We report an extensive set of robustness checks in the supplementary appendix. For example, to address the robustness of our classification of firms we show results that rely on model-based re-classification and two-sided random-effects estimation. To check sensitivity to departures from discrete heterogeneity, we show estimates of interactive regressions where worker types are not restricted to be discrete, and we report estimates that allow for within-class firm heterogeneity. We also provide evidence that our approach is helpful to alleviate biases arising from the fact that some firms have few job movers in the data. We consistently find magnitudes that are in line with our baseline estimates. In particular, in all specifications firm effects net of worker composition explain less than 5% of the log-earnings variance, and the correlation between worker and firm effects ranges between 30% and 50%.

Interpretation. The coexistence of strong sorting and weak complementarities may be surprising in the perspective of the search matching models inspired by Gary Becker. In such models, assortative matching patterns arise from complementarities in production, which are often reflected in non-monotonic wages. For example, this happens in the model of Shimer and Smith (2000) with on-the-job search that we calibrate in the supplementary appendix. More generally, the results in this section are difficult to reconcile with models based on revealed preferences for wages only. This may indicate that workers or firms care about other job attributes (Hwang et al., 1998), or that workers of a similar type enjoy working together as in the peer effects literature. Workers and firms might also be more complex than assumed in most models of sorting (Lindenlaub, 2017, Lise and Postel-Vinay, 2015). More specifically to the Swedish context, as we discuss in Appendix B, institutions such as unions may contribute to explain why different firms do not pay similar workers very differently.

6 Empirical results II: Dynamic model

In this section we present empirical results for our dynamic model. We start by describing the parameter estimates, and we then assess their implications in terms of dynamic patterns.

Table 3: Parameter estimates of the dynamic model

	Earnings effects $\xi_2(k')$ of future firm classes								
$k' =$	2	3	4	5	6	7	8	9	10
estimate	-0.005	0.004	0.005	0.022	0.002	0.015	0.009	0.016	0.023
	(0.008)	(0.009)	(0.011)	(0.012)	(0.011)	(0.010)	(0.011)	(0.012)	(0.012)
	Earnings effects $\xi_3(k)$ of past firm classes								
$k =$	2	3	4	5	6	7	8	9	10
estimate	0.051	0.038	0.045	0.061	0.040	0.072	0.058	0.087	0.090
	(0.016)	(0.015)	(0.015)	(0.016)	(0.017)	(0.015)	(0.018)	(0.016)	(0.017)
	Persistence parameters ρ								
		$\rho_{1\|2}$		$\rho^m_{3\|2}$		$\rho^s_{3\|2}$		$\rho_{4\|3}$	
		0.227		0.246		0.681		0.651	
		(0.009)		(0.044)		(0.022)		(0.004)	

Notes: Dynamic model, 2001-2005. $\rho^m_{3|2}$ is the autoregressive coefficient of log-earnings for job movers between 2002 and 2004; $\rho^s_{3|2}$ is the coefficient for job stayers. $\xi_2(k')$ and $\xi_3(k)$ are the mean effects on log-earnings before and after a job move between firm classes k and k', respectively. $k' = 1$ (resp., $k = 1$) is the omitted category. Parametric bootstrap standard errors in parentheses (200 replications).

6.1 Parameter estimates

We estimate the dynamic model on 2001-2005, focusing on males both fully-employed in the same firm in 2001-2002, and fully-employed in the same firm in 2004-2005. In order to estimate firm classes we use the same weighted k-means algorithm as in the static model. We then estimate the model in three steps, as explained in Section 4: we estimate the earnings persistence parameters $\rho_{4|3}$ and $\rho_{1|2}$, the wage functions and type probabilities of job movers, and finally those of job stayers.

Table 3 shows estimates of several parameters of the model. The parameter $\xi_2(k')$ is the effect of firm class k' on the mean log-earnings in period 2 of a worker moving from k in period 2 to k' in period 3. It would be zero for all k' under the strict exogeneity assumption on mobility, which is imposed in our static model and many models in the literature. We see that the effects are quantitatively small, with at most a 2% effect relative to the omitted class $k' = 1$. Note that strict exogeneity of mobility also imposes that earnings realizations are independent of the subsequent decision to move. This assumption cannot be tested from Table 3, but we will check its empirical plausibility in Table 4 below.

The parameter $\xi_3(k)$, in turn, captures the effect of firm class k on the mean log-earnings of a worker moving between k and k'. Our static model, and many models in the literature, would also rule out the presence of a direct effect of the previous employer on current earnings. This effect appears empirically quite large in the dynamic model. It is approximately monotonic in firm class, and amounts to a 9% effect in the highest classes. This suggests that past firms have an impact on future earnings.

In the bottom panel of Table 3 we report the estimates of earnings persistence parameters. Persistence estimates are higher for job stayers than for job movers. Notice however that the autoregressive coefficient of .246 upon job move is significantly different from zero, which suggests that the conditional independence assumption of the static model does not hold in our data. In addition, our framework delivers estimates of workers' mobility patterns among firms. In Figure S15 in the supplementary appendix we report the joint frequencies of firm classes upon job mobility for all worker types.

The estimates of the dynamic model deliver similar cross-sectional patterns for log-earnings and sorting as in the static case, as we show in the supplementary appendix (see Figure S16 and Table S13). In particular, the model implies approximate additivity of log-earnings in worker types and firm classes, relatively small differences across firms for all worker types except the lowest one, and strong evidence of association between worker types and firm classes. We report cross-sectional variance decomposition and reallocation exercises for the dynamic model, and we find similar magnitudes as in Table 2. In addition, we show that our estimates are stable across a range of specifications. As an important check, we estimate the persistence parameters based on covariance restrictions in first differences, as opposed to levels. The literature has documented differences between level estimates and first difference estimates of the dynamics of earnings in several data sets (Daly et al., 2016). We find $\rho_{1|2} = .506$, $\rho_{4|3} = .451$, $\rho^m_{3|2} = .194$, and $\rho^s_{3|2} = .605$. Despite the differences in persistence estimates, variance decomposition results are similar in this case.

6.2 Dynamic effects

While the dynamic and static models have similar cross-sectional implications, the richer setting we consider in this section allows us to study dynamic aspects of worker mobility and earnings, which are interesting from both empirical and theoretical perspectives. We start with endogenous mobility.

Table 4: Transition probabilities ($\times 100$) by conditional decile of previous earnings

	All			
	All movers	k'=1−3	k'=4−7	k'=8−10
k=1−3	2.20	0.84	1.06	0.30
	(0.03)	(0.02)	(0.03)	(0.02)
k=4−7	1.86	0.39	1.03	0.44
	(0.03)	(0.02)	(0.02)	(0.01)
k=8−10	2.90	0.47	1.00	1.43
	(0.06)	(0.06)	(0.03)	(0.02)
	First conditional decile of earnings			
	All movers	k'=1−3	k'=4−7	k'=8−10
k=1−3	3.41	1.53	1.52	0.37
	(0.21)	(0.09)	(0.12)	(0.05)
k=4−7	3.20	0.77	1.74	0.69
	(0.17)	(0.09)	(0.09)	(0.05)
k=8−10	4.92	0.93	1.74	2.25
	(0.31)	(0.15)	(0.14)	(0.13)
	Tenth conditional decile of earnings			
	All movers	k'=1−3	k'=4−7	k'=8−10
k=1−3	2.76	0.95	1.42	0.40
	(0.20)	(0.08)	(0.13)	(0.06)
k=4−7	1.82	0.35	1.05	0.42
	(0.10)	(0.04)	(0.06)	(0.03)
k=8−10	2.03	0.29	0.71	1.03
	(0.14)	(0.06)	(0.06)	(0.08)

Notes: Probability of moving, overall and by destination firm class k', for each origin firm class. Top panel: all workers. Middle and bottom panels: first and tenth decile of log-earnings Y_{i2} conditional on worker type α_i and current firm class $k_{i2} = k$. Probabilities of job mobility computed by simulation, see the supplementary appendix. Dynamic model, $10,000,000$ simulations. Parametric bootstrap standard errors in parentheses (200 replications).

Endogenous mobility. Here we study how the current wage affects a worker's decision to move, and which firm she moves to. In models where there is no match heterogeneity and mobility is efficient the wage before the move should not have an effect on the propensity to move, since a worker would always move towards higher-surplus firms irrespective of her current wage. In contrast, in wage posting models with match-specific heterogeneity, a worker

Table 5: Decomposition of the within-worker-type variance of log-earnings ($\times 100$) implied by the dynamic model

		within current firm variance	
	between current firm	between past firm	residual
year after the move (2004)	8.36 (1.09)	0.94 (0.44)	90.78 (1.33)
two year after the move (2005)	9.79 (1.25)	0.44 (0.20)	89.85 (1.36)

	between past firm variance		
	total	network effect	state dependence
year after the move (2004)	2.57 (0.75)	0.84 (0.14)	1.74 (0.67)
two year after the move (2005)	2.04 (0.49)	0.98 (0.16)	1.05 (0.39)

Notes: All numbers are in percentage of the within-type variance of log-earnings. Job movers between 2002 and 2004 only. Bottom panel: "network effect" and "state dependence" are defined in the text. Dynamic model, $1,000,000$ *simulations. Parametric bootstrap standard errors in parentheses (*200 *replications).*

is more (respectively, less) likely to accept job offers when her current match is of low (resp. high) quality. In addition, endogenous mobility is ruled out in standard AKM fixed-effects regressions.

In Table 4 we report the overall probability of a job move conditional on the firm class at origin, as well as the job move probabilities by firm class at destination. In the top panel we show those numbers in the full sample, while in the middle and bottom panels we select workers for whom the earnings rank before the move, given firm class and worker type, is below 10 or above 90 percents, respectively. In the first column we see that while the overall mobility rate lies between 2% and 3%, it is substantially higher for workers who had a low earnings realization in 2002. This suggests that workers are more likely to move when paid less. Such endogenous mobility is in line with estimates in Abowd et al. (2018), and is consistent with the predictions of wage posting models with match-specific heterogeneity. In contrast, our estimates suggest that high earnings realizations do not strongly affect mobility. Lastly, results by firm class at destination do not seem to vary much with earnings realizations, which is in line with the small estimates of $\xi_2(k')$ above.

State dependence and network effects. We now study how a worker's wage is affected by the firms she works for over time. Specifically, we ask: among similar workers in similar firms, how much of wage dispersion is explained by the workers' previous employers? Furthermore, we study two different reasons why the previous employer may matter. First, working in a high-wage firm, say, may make a worker more likely to move to another high-wage firm. We refer to this as the *network effect*. Second, the past firm may have a direct effect on the worker's wage after a job move. We call this the *state dependence effect*.

Network and state dependence effects are present in many models of earnings and mobility. Consider as an example sequential bargaining models where firms are characterized by their productivities (e.g., Postel-Vinay and Robin, 2002, Lise et al., 2016, Bagger and Lentz, 2014). Due to the on-the-job search process, the distribution of productivities of future employers depends on the productivity of the current employer (network effect). Moreover, due to the offer and counteroffer mechanism, a worker coming from a more productive firm is able to extract a higher share of the surplus from the poaching firm, compared to a worker coming from a less productive firm (state dependence). In AKM and our static model, network effects are allowed for but there is no state dependence.

To quantify these two effects, in the remainder of this section we focus on the part of the variance of log-earnings following a job move that is net of worker heterogeneity. This within-type variance amounts to 40% of the overall variance after the move. To understand how much of the log-earnings variance is explained by previous employers, in the top panel of Table 5 we show the share of variance explained by the current employer (first column) and compare it to the one explained by the previous employer holding the current firm class constant (second column). We find that, immediately after the move, the class of the previous employer contributes to .94% of the variance. This is 10% of the contribution of the class of the current firm. One year after the move (that is, in 2005) the contribution of the previous employer is twice as small (.44%), suggesting that the effect decreases over time.

We next turn to disentangling network and state dependence effects. For this, we decompose the log-earnings variance explained by the previous firm class as follows (holding worker types constant and omitting them from the notation for conciseness):

$$\begin{aligned}\mathrm{Var}\left(\mathbb{E}\left(Y_{i3}\,|\,k_{i2}\right)\right) &= \mathrm{Var}\Big(\,\mathbb{E}\Big[\mathbb{E}(Y_{i3}\,|\,k_{i3},k_{i2})\,\Big|\,k_{i2}\Big]\,\Big)\\ &= \underbrace{\mathrm{Var}\,\Big(\,\mathbb{E}\Big[\mathbb{E}(Y_{i3}\,|\,k_{i3})\,\Big|\,k_{i2}\Big]\,\Big)}_{\text{network effect}}\\ &\qquad\qquad + \underbrace{\mathrm{Var}\left(\mathbb{E}\left(Y_{i3}\,|\,k_{i2}\right)\right) - \mathrm{Var}\Big(\,\mathbb{E}\Big[\mathbb{E}(Y_{i3}\,|\,k_{i3})\,\Big|\,k_{i2}\Big]\,\Big)}_{\text{state dependence effect}}.\end{aligned}$$

Note that the first term in the decomposition ignores the direct effect that the previous firm class k_{i2} could have on the wage after the move Y_{i3}, given the current class k_{i3}. Hence the network effect measures the dependence on the past firm class k_{i2} which is solely due to the dependence of k_{i3} on k_{i2}.

In the bottom panel of Table 5 we report the results of the decomposition. The first column shows that 2.6% of the log-earnings variance immediately after the move is explained by the previous employer. One third is due to the network effect (0.8%), while two thirds reflect state dependence (1.7%). The total effect of the previous employer is 2% one year later in 2005, with a smaller relative contribution of state dependence. Hence, according to our results, at least in the short run, state dependence is of a similar order of magnitude (in fact, larger) compared to the network effect.

These findings suggest that, while it is important to study how workers move between firms, it is equally important to understand how wages are set dynamically around such moves. These issues have been studied theoretically in structural settings through the use of specific contracting environments. The results in this section should be of interest for the empirical modeling of mobility and earnings, since standard static regression models rule out state dependence while leaving network effects fully unrestricted.

7 Conclusion

In this paper we propose a framework to allow for two-sided unobserved heterogeneity in matched employer employee data sets. We introduce empirical models which allow for worker-firm interactions and dynamics, hence for mechanisms that have been emphasized in theoretical work. We provide conditions for identification in short panels, and develop estimators for finite mixture models and regression specifications.

Our application to Swedish administrative data shows that an additive model provides a good first-order approximation to the variance structure of log-earnings, while at the same time showing a strong association between worker and firm heterogeneity and a small relative contribution of firms to earnings dispersion. These findings, which are robust to a wide variety of specification checks, differ from many estimates of variance components in the literature. A recent paper by Borovickova and Shimer (2017) proposes a different measure of sorting and also finds a strong worker-firm association on Austrian data. Another recent paper by Lentz et al. (2017) uses an estimator related to ours to study wages and mobility using Danish administrative data while accounting for unemployment.

Using our dynamic model we find that endogenous mobility, by which earnings shocks affect mobility decisions, and state dependence and network effects, by which past firms have an impact on earnings after a job move, are features of our data. These findings support mechanisms that have been emphasized in the structural literature. At the same time, our estimates call for theoretical models that, unlike standard sorting models where complementaries between agents drive the nature of the allocation, can rationalize the presence of a relatively small firm effect and a strong association between worker and firm heterogeneity.

Our two-step estimation approach preserves parsimony by reducing the dimension of firm heterogeneity to a smaller number of classes, and modeling the conditional distributions of worker types. We show this strategy is helpful in alleviating small-sample biases arising from low mobility rates. In companion work (Bonhomme et al., 2017) we further study the theoretical properties of approaches based on an initial clustering step, viewing discrete estimation as an approximation to individual or firm heterogeneity.

Two-step estimation could be useful in structural settings too, where joint estimation of the distribution of two-sided heterogeneity and the structural parameters may be computationally prohibitive. An attractive feature is that the classification does not rely on the entire model's structure, solely on the fact that unobserved firm heterogeneity operates at the class level. Our identification results could also prove useful for structural models of workers and firms. In this light, an interesting extension of our results would be to allow for time-varying processes of worker types that could vary in response to firm-level shocks.

Lastly, this paper proposes a portable methodology for empirical work. Our methods may reveal interesting patterns of sorting and complementarities in other studies of workers and firms, including in relatively small samples such as a particular occupation or a short period of time (e.g., around a recession), where dimension reduction is likely to be particularly helpful. More generally, we expect our methods to be useful in other settings involving matched panel data, for example in economics of education, urban economics, or finance.

References

Abowd, J. M., R. H. Creecy, and F. Kramarz (2002): "Computing Person and Firm Effects Using Linked Longitudinal Employer-Employee Data," *Unpublished Manuscript.*

Abowd, J. M., F. Kramarz, P. Lengermann, and S. Pérez-Duarte (2004): "Are Good Workers Employed by Good Firms? A Test of a Simple Assortative Matching Model for France and the United States," *Unpublished Manuscript.*

Abowd, J. M., F. Kramarz, and D. N. Margolis (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.

Abowd, J. M., K. L. McKinney, and I. M. Schmutte (2018): "Modeling endogenous mobility in earnings determination," *Journal of Business & Economic Statistics*, 1–14.

Airoldi, E. M., D. M. Blei, S. E. Fienberg, and E. P. Xing (2008): "Mixed Membership Stochastic Blockmodels," *J. Mach. Learn. Res.*, 9, 1981–2014.

Akerman, A., E. Helpman, O. Itskhoki, M.-A. Muendler, and S. Redding (2013): "Sources of Wage Inequality," *American Economic Review, Papers and Proceedings.*, 103, 214–219.

Andrews, M. J., L. Gill, T. Schank, and R. Upward (2008): "High Wage Workers and Low Wage Firms: Negative Assortative Matching or Limited Mobility Bias?" *J. R. Stat. Soc. Ser. A Stat. Soc.*, 171, 673–697.

——— (2012): "High Wage Workers Match with High Wage Firms: Clear Evidence of the Effects of Limited Mobility Bias," *Econ. Lett.*, 117, 824–827.

Bagger, J. and R. Lentz (2014): "An Empirical Model of Wage Dispersion with Sorting," *University of Wisconsin mimeo.*

Barth, E., A. Bryson, J. C. Davis, and R. Freeman (2014): "It's Where You Work: Increases in Earnings Dispersion Across Establishments and Individuals in the U.S." *Unpublished Manuscript.*

Bartolucci, C., F. Devicienti, and I. Monzon (2015): "Identifying Sorting in Practice," *Unpublished Manuscript.*

Baumgarten, Daniel, G. F. and S. Lehwald (2014): "Drivers of Wage Inequality in Germany: Trade, Technology, or Institutions?" *Unpublished Manuscript.*

Becker, G. S. (1973): "A Theory of Marriage: Part I," *Journal of Political Economy*, 81, 813–46.

Blei, D. M., A. Y. Ng, and M. I. Jordan (2003): "Latent Dirichlet Allocation," *J. Mach. Learn. Res.*, 3, 993–1022.

Bonhomme, S. (2017): "Econometric Analysis of Bipartite Networks," *to appear in "The Econometric Analysis of Network data," edited by B. Graham and A. De Paula.*

Bonhomme, S., K. Jochmans, and J. M. Robin (2014): "Nonparametric Estimation of Finite Mixtures from Repeated Measurements," *Journal of the Royal Statistical Society, Series B, forthcoming.*

Bonhomme, S., T. Lamadon, and E. Manresa (2017): "Discretizing Unobserved Heterogeneity," *Unpublished Manuscript.*

Bonhomme, S. and E. Manresa (2015): "Grouped Patterns of Heterogeneity in Panel Data," *Econometrica*, 83, 1147–1184.

Borovickova, K. and R. Shimer (2017): "High Wage Workers Work for High Wage Firms," *Unpublished Manuscript.*

Burdett, K. and M. G. Coles (2003): "Wage/Tenure Contracts and Equilibrium Search," *Econometrica*, 71, 1377–1404.

Burdett, K. and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *Int. Econ. Rev.*, 257–273.

Card, D., J. Heining, and P. Kline (2013): "Workplace Heterogeneity and the Rise of West German Wage Inequality," *Q. J. Econ.*, 128, 967–1015.

Daly, M., D. Hryshko, and I. Manovskii (2016): "Improving the Measurement of Earnings Dynamics," *Unpublished Manuscript.*

Delacroix, A. and S. Shi (2006): "Directed Search on the Job and the Wage Ladder," *Int. Econ. Rev.*, 47, 651–699.

Dempster, A. P., N. M. Laird, and D. B. Rubin (1977): "Maximum Likelihood from Incomplete Data via the EM Algorithm," *J. R. Stat. Soc. Series B Stat. Methodol.*, 39, 1–38.

Eeckhout, J. and P. Kircher (2011): "Identifying Sorting in Theory," *Rev. Econ. Stud.*, 78, 872–906.

Finkelstein, A., M. Gentzkow, and H. Williams (2016): "Sources of Geographic Variation in Health Care: Evidence from Patient Migration," *Quarterly Journal of Economics*, 131, 1681–1726.

Fitzenberger, B. and A. Garloff (2007): "Descriptive Evidence on Labor Market Transitions and the Wage Structure in Germany," *Int. Econ. Rev.*, 227, 115–152.

Friedrich, B., L. Laun, C. Meghir, and L. Pistaferri (2014): "Earnings Dynamics and Firm-Level Shocks," *Unpublished Manuscript*.

Goldschmidt, D. and J. Schmieder (2015): "The Rise of Domestic Outsourcing and the Evolution of the German Wage Structure," *to appear in Quarterly Journal of Economics*.

Graham, B. S., G. W. Imbens, and G. Ridder (2014): "Complementarity and Aggregate Implications of Assortative Matching: A Nonparametric Analysis," *Quantitative Economics*, 5, 29–66.

Gruetter, M. and R. Lalive (2009): "The Importance of Firms in Wage Determination," *Labour Econ.*, 16, 149–160.

Hagedorn, M., T. H. Law, and I. Manovskii (2017): "Identifying equilibrium models of labor market sorting," *Econometrica*, 85, 29–65.

Hahn, J. and H. R. Moon (2010): "Panel Data Models with Finite Number of Multiple Equilibria," *Econometric Theory*, 26, 863–881.

Hall, P. and X.-H. Zhou (2003): "Nonparametric Estimation of Component Distributions in a Multivariate Mixture," *Ann. Stat.*, 31, 201–224.

Hall, R. E. and F. S. Mishkin (1982): "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," *Econometrica*, 50, 461–481.

Henry, M., Y. Kitamura, and B. Salanié (2014): "Partial Identification of Finite Mixtures in Econometric Models," *Quant. Econom.*, 5, 123–144.

Hu, Y. (2008): "Identification and Estimation of Nonlinear Models with Misclassification Error Using Instrumental Variables: A General Solution," *Journal of Econometrics*, 144, 27–61.

Hu, Y. and S. M. Schennach (2008): "Instrumental Variable Treatment of Nonclassical Measurement Error Models," *Econometrica*, 76, 195–216.

Hu, Y. and M. Shum (2012): "Nonparametric Identification of Dynamic Models with Unobserved State Variables," *J. Econom.*, 171, 32–44.

Hwang, H.-s., D. T. Mortensen, and W. R. Reed (1998): "Hedonic wages and labor market search," *Journal of Labor Economics*, 16, 815–847.

Jackson, C. K. (2013): "Match Quality, Worker Productivity, and Worker Mobility: Direct Evidence from Teachers," *Rev. Econ. Stat.*, 95, 1096–1116.

Jochmans, K. and M. Weidner (2017): "Fixed-Effect Regressions on Network data," *Unpublished Manuscript*.

Kline, P., R. Saggio, and M. Sølvsten (2018): "Leave-out estimation of variance components," *ArXiv e-prints*.

Kramarz, F., S. J. Machin, and A. Ouazad (2015): "Using Compulsory Mobility to Identify School Quality and Peer Effects," *Oxford Bulletin of Economics and Statistics*, 77, 566–587.

Lamadon, T., J. Lise, C. Meghir, and J. M. Robin (2013): "Matching, Sorting, Firm Productivity and Wages," *Unpublished Manuscript*.

Lentz, R., S. Piyapromdee, and J.-M. Robin (2017): "On Worker and Firm Heterogeneity in Wages and Employment Mobility: Evidence from Danish Register Data," *Unpublished Manuscript*.

Levine, M., D. R. Hunter, and D. Chauveau (2011): "Maximum Smoothed Likelihood for Multivariate Mixtures," *Biometrika*, 98, 403–416.

Lin, C.-C. and S. Ng (2012): "Estimation of Panel Data Models with Parameter Heterogeneity when Group Membership is Unknown," *Journal of Econometric Methods*, 1.

Lindenlaub, I. (2017): "Sorting multidimensional types: Theory and application," *The Review of Economic Studies*, 84, 718–789.

Lise, J., C. Meghir, and J.-M. Robin (2016): "Matching, sorting and wages," *Review of Economic Dynamics*, 19, 63–87.

Lise, J. and F. Postel-Vinay (2015): "Multidimensional skills, sorting, and human capital accumulation," *Manuscript, University of Minnesota*.

Lise, J. and J.-M. Robin (2013): "The Macro-Dynamics of Sorting Between Workers and Firms," *to appear in American Economic Review*.

Lopes de Melo, R. (2018): "Firm wage differentials and labor market sorting: Reconciling theory and evidence," *Journal of Political Economy*, 126, 313–346.

Mendes, R., G. J. van den Berg, and M. Lindeboom (2010): "An Empirical Assessment of Assortative Matching in the Labor Market," *Labour Econ.*, 17, 919–929.

Postel-Vinay, F. and J.-m. Robin (2002): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, 2295–2350.

Reiersøl, O. (1950): "Identifiability of a Linear Relation between Variables Which Are Subject to Error," *Econometrica*, 18, 375–389.

Shi, S. (2008): "Directed Search for Equilibrium Wage-Tenure Contracts," *Econometrica*, 77, 561–584.

Shimer, R. (2005): "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 113, 996–1025.

Shimer, R. and L. Smith (2000): "Assortative Matching and Search," *Econometrica*, 68, 343–369.

Skans, O. N., P.-A. Edin, and B. Holmlund (2009): "Wage Dispersion Between and Within Plants: Sweden 1985-2000," in *The Structure of Wages: An International Comparison*, ed. by Edward P. Lazear and Kathryn L. Shaw, University of Chicago Press, 217–260.

Song, J., D. J. Price, F. Guvenen, and N. Bloom (2015): "Firming up Inequality," *Unpublished Manuscript*.

Sorkin, I. (2018): "Ranking firms using revealed preference," *The quarterly journal of economics*, 133, 1331–1393.

Steinley, D. (2006): “K-means Clustering: a Half-Century Synthesis,” *Br. J. Math. Stat. Psychol.*, 59, 1–34.

Woodcock, S. D. (2008): “Wage Differentials in the Presence of Unobserved Worker, Firm, and Match Heterogeneity,” *Labour Econ.*, 15, 771–793.

APPENDIX

A Proof of Theorem 1

Let $k \in \{1, ..., K\}$, and let $(k_1, ..., k_R)$, $(\widetilde{k}_1, ..., \widetilde{k}_R)$ as in Assumption 3, with $k_1 = k$. From (7) we have, considering workers who move from k_r to $\widetilde{k}_{r'}$ for some $r \in \{1, ..., R\}$ and $r' \in \{r-1, r\}$:

$$\Pr\left[Y_{i1} \leq y_1, Y_{i2} \leq y_2 \,|\, k_{i1} = k_r, k_{i2} = \widetilde{k}_{r'}, m_{i1} = 1\right] = \sum_{\alpha=1}^{L} p_{k_r,\widetilde{k}_{r'}}(\alpha) F_{k_r,\alpha}(y_1) F^m_{\widetilde{k}_{r'},\alpha}(y_2). \tag{A1}$$

Consider sets of M values for y_1 and y_2 that satisfy Assumption 3 *ii*). Note that one can augment those sets with a finite number of other values, including $+\infty$, while preserving the rank condition in Assumption 3 *ii*). Writing (A1) in matrix notation we obtain:

$$A(k_r, \widetilde{k}_{r'}) = F(k_r) D(k_r, \widetilde{k}_{r'}) F^m(\widetilde{k}_{r'})^\intercal, \tag{A2}$$

where $A(k_r, \widetilde{k}_{r'})$ is $M \times M$ with generic element:

$$\Pr\left[Y_{i1} \leq y_1, Y_{i2} \leq y_2 \,|\, k_{i1} = k_r, k_{i2} = \widetilde{k}_{r'}, m_{i1} = 1\right],$$

$F(k_r)$ is $M \times L$ with element $F_{k_r,\alpha}(y_1)$, $F^m(\widetilde{k}_{r'})$ is $M \times L$ with element $F^m_{\widetilde{k}_{r'},\alpha}(y_2)$, $D(k_r, \widetilde{k}_{r'})$ is $L \times L$ diagonal with element $p_{k_r,\widetilde{k}_{r'}}(\alpha)$, and $A^\intercal$ denotes the transpose of matrix A.

Note that $A(k_r, \widetilde{k}_{r'})$ has rank L by Assumption 3 *ii*). Consider a singular value decomposition of $A(k_1, \widetilde{k}_1)$:

$$A(k_1, \widetilde{k}_1) = F(k_1) D(k_1, \widetilde{k}_1) F^m(\widetilde{k}_1)^\intercal = USV^\intercal,$$

where S is $L \times L$ diagonal and non-singular, and U and V have orthonormal columns. We define the following matrices:

$$\begin{aligned} B(k_r, \widetilde{k}_{r'}) &= S^{-\frac{1}{2}} U^\intercal A(k_r, \widetilde{k}_{r'}) V^\intercal S^{-\frac{1}{2}}, \\ Q(k_r) &= S^{-\frac{1}{2}} U^\intercal F(k_r). \end{aligned}$$

$B(k_r, \widetilde{k}_{r'})$ and $Q(k_r)$ are non-singular by Assumption 3 *ii*). Moreover, we have, for all $r \in \{1, ..., R\}$:

$$\begin{aligned} B(k_r, \widetilde{k}_r)B(k_{r+1}, \widetilde{k}_r)^{-1} &= S^{-\frac{1}{2}}U^\intercal A(k_r, \widetilde{k}_r)V^\intercal S^{-\frac{1}{2}} \left(S^{-\frac{1}{2}}U^\intercal A(k_{r+1}, \widetilde{k}_r)V^\intercal S^{-\frac{1}{2}}\right)^{-1} \\ &= S^{-\frac{1}{2}}U^\intercal F(k_r)D(k_r, \widetilde{k}_r)\left(S^{-\frac{1}{2}}U^\intercal F(k_{r+1})D(k_{r+1}, \widetilde{k}_r)\right)^{-1} \\ &= Q(k_r)D(k_r, \widetilde{k}_r)D(k_{r+1}, \widetilde{k}_r)^{-1}Q(k_{r+1})^{-1}. \end{aligned}$$

Let $E_r = B(k_r, \widetilde{k}_r)B(k_{r+1}, \widetilde{k}_r)^{-1}$. We thus have:

$$E_1E_2...E_R = Q(k_1)D(k_1, \widetilde{k}_1)D(k_2, \widetilde{k}_1)^{-1}...D(k_R, \widetilde{k}_R)D(k_1, \widetilde{k}_R)^{-1}Q(k_1)^{-1}.$$

The eigenvalues of this matrix are all distinct by Assumption 3 *i*), so $Q(k_1) = Q(k)$ is identified up to right-multiplication by a diagonal matrix and permutation of its columns.

Now, note that $F(k) = UU^\intercal F(k)$, so:

$$F(k) = US^{\frac{1}{2}}Q(k)$$

is identified up to right-multiplication by a diagonal matrix and permutation of its columns. Hence $F_{k\alpha}(y_1)\lambda_\alpha$ is identified up to a choice of labeling, where $\lambda_\alpha \neq 0$ is a scale factor. As pointed out above, without loss of generality we can assume that the set of y_1 values contains $y_1 = +\infty$. This implies that λ_α is identified, so $F_{k\alpha}(y_1)$ is identified up to labeling. As a result, $F_{k,\sigma(\alpha)}(y_1)$ is identified for some permutation $\sigma : \{1, ..., L\} \to \{1, ..., L\}$. To identify $F_{k,\sigma(\alpha)}$ at a point y different from the grid of M values considered so far, simply augment the set of values with y as an additional value, and apply the above arguments.

Let now $k' \neq k$, and let $(k_1, ..., k_R)$, $(\widetilde{k}_1, ..., \widetilde{k}_R)$, be an alternating cycle such that $k_1 = k$ and $k' = k_r$ for some r, by Assumption 3 *i*). We have:

$$A(k, \widetilde{k}_1) = F(k)D(k, \widetilde{k}_1)F^m(\widetilde{k}_1)^\intercal.$$

As $F_{k,\sigma(\alpha)}$ is identified and $F(k)$ has rank L:

$$p_{k,\widetilde{k}_1}(\sigma(\alpha))F^m_{\widetilde{k}_1,\sigma(\alpha)}(y_2)$$

is identified, so by taking $y_2 = +\infty$, both $p_{k,\widetilde{k}_1}(\sigma(\alpha))$ and $F^m_{\widetilde{k}_1,\sigma(\alpha)}$ are identified. Next we have:

$$A(k_2, \widetilde{k}_1) = F(k_2)D(k_2, \widetilde{k}_1)F^m(\widetilde{k}_1)^\intercal,$$

so, using similar arguments, $p_{k_2,\widetilde{k}_1}(\sigma(\alpha))$ and $F_{\widetilde{k}_2,\sigma(\alpha)}$ are identified. By induction, $p_{k_r,\widetilde{k}_{r'}}(\sigma(\alpha))$, $F_{k_r,\sigma(\alpha)}$, and $F^m_{\widetilde{k}_{r'},\sigma(\alpha)}$ are identified for all r and $r' \in \{r-1, r\}$. As $k' = k_r$, it follows that $F_{k',\sigma(\alpha)}$ are identified. Moreover, for each k' (possibly equal to k), using an alternating cycle as in the second part of Assumption 3 *i*) we obtain by a similar argument that $F^m_{k',\sigma(\alpha)}$ is identified.

Then, let $(k,k') \in \{1,...,K\}^2$. From:

$$A(k,k') = F(k)D(k,k')F^m(k')^\intercal,$$

and, from the fact that $F_{k,\sigma(\alpha)}$ and $F^m_{k',\sigma(\alpha)}$ are both identified, and that $F(k)$ and $F^m(k')$ have rank L by Assumption 3 *ii*), it follows that $p_{kk'}(\sigma(\alpha))$ is identified.

To show the last part of Theorem 1, note that by the first part of the proof there exists a permutation $\sigma : \{1,...,L\} \to \{1,...,L\}$ such that $F_{k,\sigma(\alpha)}$ is identified for all k, α. Now we have, writing (8) for the L worker types and M values of y_1 given by Assumption 3 *ii*) in matrix form:

$$H(k) = F(k)P(k),$$

where $H(k)$ has generic element $\Pr[Y_{i1} \leq y_1 \,|\, k_{i1} = k]$, the $L \times 1$ vector $P(k)$ has generic element $q_k(\sigma(\alpha))$, and the columns of $F(k)$ have been ordered with respect to σ. By Assumption 3 *ii*), $F(k)$ has rank L, from which it follows that:

$$P(k) = \left[F(k)^\intercal F(k)\right]^{-1} F(k)^\intercal H(k)$$

is identified. So $q_k(\sigma(\alpha))$ is identified.

B Data

We use a match of four different databases from Friedrich et al. (2014) covering the entire working age population in Sweden between 1997 and 2008. The Swedish data registry (ANST), which is part of the register-based labor market statistics at Statistics Sweden (RAMS), provides information about individuals, their employment, and their employers. This database is collected yearly from the firm's income statements. The other databases provide additional information on worker and firm characteristics, as well as unemployment status of workers: LOUISE/LINDA contains information on the workers, SBS provides accounting data and balance sheet information for all non-financial corporations in Sweden, and the Unemployment Register contains spells of unemployment registered at the Public Employment Service.

The RAMS dataset allows constructing individual employment spells within a year, as it provides the first and last remunerated month for each employee in a plant as well as firm and plant identifier. We define firms through firm identifiers. We define the main employment of each individual in a year as the one providing the highest earnings in that year. The main employer determines the employer of a worker in a given year. RAMS provides pre-tax yearly earnings for each spell. We use the ratio between total earnings at the main employer and the number of months employed as our measure of monthly earnings. We compute real earnings in 2007 prices.

Sample selection. Following Friedrich et al. (2014) we perform a first sample selection by dropping all financial corporations and some sectors such as fishery and agriculture, education, health and social work. In addition, all workers from the public sector or self-employed are discarded. We focus on workers employed in years 2002 and 2004. These two years correspond to periods 1 and 2 in the static model. We restrict the sample to males. We choose not to include female workers in the analysis in order to avoid dealing with gender differences in labor supply, since we do not have information on hours worked. We keep firms which have at least one worker who is fully employed in both 2002 and 2004 ("continuing firms"), where fully employed workers are those employed in all twelve months in a year in one firm. From this 2002-2004 sample we define the sub-sample of movers as workers whose firm identifier changes between 2002 and 2004. If a worker returns in 2004 to the firm she worked for in 2002 we do not consider this worker to be a mover (4.3% of the sample).

Restricting workers to be fully employed in 2002 and 2004, and firms to be present in both periods, is not innocuous, and we will see that this results in a substantial reduction of the number of workers whose firm identifier changes in the course of 2003. The reason for this conservative sample selection is that we want to capture, as closely as possible, individual job moves between existing firms. In particular, a firm may appear in only one period because of a merger or acquisition, entry or exit, or due to a re-definition of the firm identifier over time. Although we have conducted robustness checks, in our preferred specification we do not include these job moves as we do not think that they map naturally to our model. For the dynamic model we consider a subsample that covers the years 2001 to 2005. In addition to the criteria used to construct the 2002-2004 sample, we require that workers be fully employed in the same firm in 2001 and 2002, and in 2004 and 2005.

Descriptive Statistics We now report descriptive statistics on the 2002-2004 and 2001-2005 samples, as well as on the subsamples of job movers. Figures can be found in Table B1. The 2002-2004 sample contains about 600,000 workers and 44,000 firms. Hence the average number of workers per firm is 13.7. The mean firm size as reported by the firm is higher, 37.6, due to our sample selection that focuses on fully employed male workers. In the 2001-2005 sample, the mean number of workers and mean reported size are 12.3 and 37.1, respectively. The distribution of firm size is skewed, and medians are smaller. At the same time, reported firm sizes in the subsamples of movers are substantially higher.

Identification relies on workers moving between firms over time. In the 2002-2004 sample, the mobility rate, which we define as the fraction of workers fully employed in 2002 and 2004 whose firm identifiers are different in these two years, is 19557/599775 = 3.3%. In the 2001-2005 sample the rate is 2.4%. These numbers are lower than the ones calculated by Skans et al. (2009), who document between-plant mobility rates ranging between 4% and 6% between 1986 and 2000.[16] To understand

[16] See their Figure 7.14. Skans et al. (2009) report the fraction of workers employed in plants with more than

how our sample selection influences the mobility rate, we have computed similar descriptive statistics on the entire 2002-2004 sample, without imposing that workers are fully employed in 2002 and 2004 or that firms exist in the two periods, see Table S2 in the supplementary appendix. Removing the requirements of full-year employment in both 2002 and 2004 and continuously existing firms results in a considerably less restrictive definition of mobility, as the mobility rate is 11.2% in this case.[17] Although we prefer to focus on a more restrictive definition for estimation, as a robustness check we have also estimated the models on this larger sample, finding comparable results.

The between-firm log-earnings variance represents 38.3% of total log-earnings variance in 2002. This number is higher than the 31% percentage explained between plants in 2000, as reported by Skans et al. (2009). However, despite growing steadily over the past decades, the between-firm (or plant) component is still lower compared with other economies such as Germany, Brazil, or the US. In Germany and Brazil, between components are closer to 50%, see Baumgarten and Lehwald (2014) or Akerman et al. (2013), for example. In the US, Barth et al. (2014) report a between-establishment log-earnings component of 46% to 49% in 1996-2007.

While differences across countries need to be interpreted cautiously due to differences in earnings definition or data collection, lower levels of between-firm earnings dispersion in Sweden are often attributed to historically highly unionized labor market and the presence of collective wage bargaining agreements. In particular, after World War II the introduction of the so-called solidarity wage policy, which had as guiding principle "equal pay for equal work", significantly limited the capacity of firms to differentially pay their employees. However, several reforms over the last two decades have contributed to an increase in between-firm wage variation due to a more informal coordination in wage setting (see Skans et al., 2009, and Akerman et al., 2013). It is important to keep these features of the Swedish labor market in mind when interpreting our results.

Finally, comparing the first two columns (or the last two columns) of Table B1 shows that job movers are on average younger and more educated than workers who remain in the same firm. They also tend to work more in service sectors as opposed to manufacturing. In the last row we also see that firms with a non-zero fraction of job movers seem more productive, as their value added per worker is higher. At the same time, characteristics of job movers and stayers show substantial overlap.

25 employees in years $t-1$ and t who changed plant between $t-1$ and t.

[17] As a comparison, for Germany Fitzenberger and Garloff (2007) report yearly between-employers transition rates of 7.5% in the period 1976 to 1996 for male workers.

Table B1: Data description

years:	2002-2004 all	2002-2004 movers	2001-2005 all	2001-2005 movers
number of workers	599,775	19,557	442,757	9,645
number of firms	43,826	7,557	36,928	4,248
number of firms ≥ 10	23,389	6,231	20,557	3,644
number of firms ≥ 50	4,338	2,563	3,951	1,757
mean firm reported size	37.59	132.33	39.67	184.77
median firm reported size	10	28	11	36
% high school drop out	20.6%	14%	21.5%	14.7%
% high school graduates	56.7%	57.3%	57%	59%
% some college	22.7%	28.7%	21.4%	26.3%
% workers younger than 30	16.8%	28%	13.9%	23.8%
% workers between 31 and 50	57.2%	59%	59.4%	62.1%
% workers older than 51	26%	13%	26.7%	14.2%
% workers in manufacturing	45.4%	35.1%	48.5%	40.4%
% workers in services	25.3%	33.7%	22.4%	27.8%
% workers in retail and trade	16.7%	20.3%	16.3%	20.8%
% workers in construction	12.6%	10.9%	12.8%	11%
mean log-earnings	10.18	10.17	10.19	10.17
variance of log-earnings	0.124	0.166	0.113	0.148
between-firm variance of log-earnings	0.0475	0.1026	0.0441	0.0947
mean log-value-added per worker	15.3	16.35	15.37	16.63

Notes: Swedish registry data. Males, fully employed in the same firm in 2002 and 2004 (columns 1 and 2), and fully employed in the same firm in 2001-2002 and 2004-2005 (columns 3 and 4), continuously existing firms. Figures for 2002. Mean log value added per worker reported for firms with positive value added (98.7% of firms in the 2002-2004 sample).